Estimation The mean vector and Covariance Matrix :-

There are many methods for the Parameter estimation which they are :-

(1) Maximum Likelihood Method

(2) The moment method

(3) Last squares Method.

(4) Minimum chie- square Method

(5) Bayesian Method.

We will use a method maximum Likelihood for estimation the mean vector and Covariana matrix

Maximum likelihood Method:

Suppose that $\underline{x}_1, \underline{x}_2, \dots \underline{x}_n$ be ariys taken from $N_p(\mu, \Sigma)$. The P.d.f of random vector observation $\underline{x}_i$ is: $f(\underline{x}_i) = \frac{1}{(2\pi)^{p/2}(|\Sigma|)^{1/2}} e^{-\frac{1}{2}(\underline{x}_i - \mu)' \Sigma^{-1} (\underline{x}_i - \mu)}$

$i = 1, 2, \dots n$

رقم المتغير — رقم المشاهدة

$$\underline{X}_{1\,(p\times 1)} = \begin{pmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{p1} \end{pmatrix} \quad \text{Length, weight}$$

$$\underline{X}_{2\,p\times 1} = \begin{pmatrix} x_{12} \\ x_{22} \\ \vdots \\ x_{p2} \end{pmatrix} \quad \cdots \quad \underline{X}_{p\,p\times 1} = \begin{pmatrix} x_{1p} \\ x_{2p} \\ \vdots \\ x_{pp} \end{pmatrix}$$

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The Sample mean vector $\underline{\bar{x}}$ is defind as:

$$\underline{\bar{x}} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_p \end{pmatrix} = \begin{pmatrix} \frac{\sum_{i=1}^{n} x_{1i}}{n} \\ \frac{\sum_{i=1}^{n} x_{2i}}{n} \\ \vdots \\ \frac{\sum_{i=1}^{n} x_{pi}}{n} \end{pmatrix} = \frac{1}{n} \begin{pmatrix} \sum_{i=1}^{n} x_{1i} \\ \sum_{i=1}^{n} x_{2i} \\ \vdots \\ \sum_{i=1}^{n} x_{pi} \end{pmatrix}$$

تعرفت ان المتجه العشوائي $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$ سوف يتم تغير كل $\underline{\mu}$ و Σ وصفوفة التباين Σ وذلك بسبب حجم عوائق ذات حجم n و نحتاج اعادة حيث ان متجه $\underline{x_i} \sim N_p(\underline{\mu}, \Sigma)$ $i = 1, 2, \ldots, n$ وصفوفة سوف يتم طريقة الامكانية الاعظم لتقدير متجه المتوسطات وصفوفة التباين.

Definition: likelihood function is ajoint p.d.f of random observations

$$L(\underline{\mu}, \Sigma) = f(\underline{x_1}, \underline{x_2}, \ldots, \underline{x_n})$$

$$= \begin{cases} \prod_{i=1}^{n} f(\underline{x_i}) & \text{if } x_1, x_2, \ldots x_n \text{ (indep.)} \\ f(x_1) f(x_2|x_1) f(x_3|x_1, x_2) \cdots f(x_n|x_1, x_2 \ldots x_{n-1}) & \text{(dependent)} \end{cases}$$

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حيث ان المشاهدات مستقلة بعضها عن بعض لذا دالة الامكان
تمثل دالة كثافة الاحتمال المشتركة لمتغيرات مشاهدات العينة
العشوائية التي تسحب بالشكل الاتي:-

$$L(\underline{\mu}, \underline{\Sigma}) = P(\underline{x}_1, \underline{x}_2 \dots, \underline{x}_n) = \prod_{i=1}^{n} f(\underline{x}_i)$$

$$= \prod_{i=1}^{n} \frac{1}{(2\pi)^{p/2} |\underline{\Sigma}|^{\frac{1}{2}}} e^{-\frac{1}{2}(\underline{x}_i - \underline{\mu})' \underline{\Sigma}^{-1} (\underline{x}_i - \underline{\mu})}$$

we will study the following three cases :-

(1) when $\underline{\Sigma}$ know:

$$L(\underline{\mu}) = (2\pi)^{\frac{-np}{2}} |\underline{\Sigma}|^{-\frac{n}{2}} e^{-\frac{1}{2}\sum_{i=1}^{n}(\underline{x}_i - \underline{\mu})' \underline{\Sigma}^{-1} (\underline{x}_i - \underline{\mu})}$$

---- (1)

نأخذ ln للطرفين

$$\ln L(\underline{\mu}) = \frac{-np}{2} \ln(2\pi) - \frac{n}{2} \ln |\underline{\Sigma}| - \frac{1}{2} \sum_{i=1}^{n} (x_i - \underline{\mu})' \underline{\Sigma}^{-1} (x_i - \underline{\mu})$$

نشتق المعادلة اعلاه بالنسبة لـ $\underline{\mu}$ ثم نساوي بالصفر المتجه $[\underline{0}]$

$$\frac{\partial \ln L(\underline{\mu})}{\partial \underline{\mu}} = -\frac{1}{2} \sum_{i=1}^{n} \underline{\Sigma}^{-1} (x_i - \underline{\mu}) = \underline{0} \quad \text{---- (2)}$$

$$\Rightarrow \frac{1}{2} \sum_{i=1}^{n} \underline{\Sigma}^{-1} (x_i - \underline{\mu}) = \underline{0} \qquad (* (2))$$

$$\Rightarrow \underline{\Sigma}^{-1} \sum_{i=1}^{n} (\underline{x}_i - \underline{\mu}) = \underline{0} \quad \text{---- (3)}, \qquad \bar{\underline{x}} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

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بما ان الصورة $\frac{\partial}{\partial \underline{\mu}}$ صفوفة غير متغيرة (لا تعتمد) نضرب المعادلة (3) بـ $\frac{\partial}{\partial \underline{\mu}}$ وناخذ (المجموع) ان الحالة الاخيرة

$$\Rightarrow \sum_{i=1}^{n} \underline{X}_i - n\underline{\mu} = 0$$

$$\Rightarrow n\bar{\underline{X}} - n\underline{\mu} = 0$$

$$\therefore \hat{\underline{\mu}} = \bar{\underline{X}} = \begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \vdots \\ \bar{X}_p \end{bmatrix}$$

ملاحظة:

$$\frac{\partial}{\partial \underline{X}} (\underline{a} - \underline{X})' A (\underline{a} - \underline{X}) = -A (\underline{a} - \underline{X})$$

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