(2) False-Position method (Regula falsi method):

Suppose a continuous function f defined on the interval [a,b] is given with f(a) and f(b) of opposite sign (i.e. $f(a)\times f(b)<0$). To derive a formula for false-position method, approximate the graph of the function f(x) by a straight line on [a,b] connecting (a, f(a)) and (b, f(b)) which intersect x-axis at (c,0) where c is more approximate to the exact (actual) root than a and b.

To obtain a formula for c, we use the slop equality:

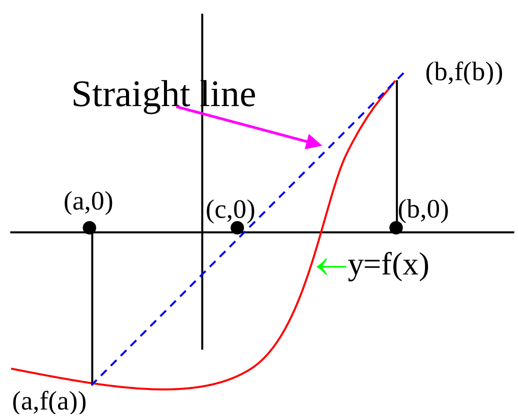
$$\frac{f(b)-f(a)}{b-a}=\frac{f(b)-y}{b-x}\Rightarrow\frac{f(b)-f(a)}{b-a}=\frac{f(b)-0}{b-c}$$

$$\Rightarrow (f(b)-f(a))(b-c)=f(b)(b-a)$$

$$\Rightarrow c=b-\frac{f(b)(b-a)}{f(b)-f(a)}$$

To find another approximation:

$$\text{If } f(a)\times f(c)\begin{cases} <0 & \text{there is a root between a, c} \Rightarrow d=c-\dfrac{f(c)(c-a)}{f(c)-f(a)} \\ >0 & \text{there is a root between c, b} \Rightarrow d=b-\dfrac{f(b)(b-c)}{f(b)-f(c)} \\ =0 & \text{c is exact root ((Stop)).} \end{cases}$$



Example (7):

Find an approximate root of the equation $x sin(x) - 1 = 0$ in the interval [0,2] by using False position method.

Solution: It is possible to use False position method because f(x) is continuous on [0,2] and f(a)=f(0)=-1; f(b)=f(2)=0.81859 $\Rightarrow$ f(a)×f(b) <0.

$$\Rightarrow c_1 = b - \frac{f(b)(b-a)}{f(b)-f(a)} = 2 - \frac{f(2)(2-0)}{f(2)-f(0)} = 2 - \frac{(0.81859)(2)}{0.81859-(-1)} = 1.0997502$$

$f(c_1)=f(1.0997502)=-0.0200192$

$\Rightarrow f(c_1)\times f(a) > 0 \Rightarrow$ there is a root between c_1 and b $\Rightarrow [c_1,b]$

$$\Rightarrow c_2 = 2 - \frac{f(2)(2-1.0997502)}{f(2)-f(1.0997502)} = 2 - 0.878759 = 1.1212407$$

$f(c_2)=f(1.1212407)=0.0098345$

$\Rightarrow f(c_2)\times f(c_1) < 0 \Rightarrow$ there is a root between c_1 and c_2 $\Rightarrow [c_1,c_2]$

$\Rightarrow c_3=1.1141612$

$f(c_3)=f(1.1141612)= 0.0000056$

$\Rightarrow f(c_3)\times f(c_1) < 0 \Rightarrow$ there is a root between c_1 and c_3 $\Rightarrow [c_1,c_3]$

$\Rightarrow c_4=1.1141571$

$f(c_4)=f(1.1141571)= 0$

Stop iteration with $c_4=1.1141571$ is the root.

Algorithm (False-Position method)

Input: a, b, ε, δ, f(x)

Step(1): If f(a)×f(b) >0 then stop (does not exist root).

Step(2): Put i=1 compute dx= $\frac{f(b)(b-a)}{f(b)-f(a)}$ and Set c= $b-dx$, and find f(c).

Step(3): If f(c)=0 then print c is a root and stop.

Step(4): else if f(a)×f(c) >0 then a=c and f(a)=f(c)

Step(5): If f(a)×f(c) <0 then b=c and f(b)=f(c)

Step(6): Evaluate if $|dx| > \varepsilon$ or $|f(c)| > \delta$ then go to step(2) with i=i+1.

Step(7): Print c

<u>Home work:</u>

Find an approximate root of f(x)=x ln(x)-1=0 in the interval [1,2] by using False-Position method with error $\leq \varepsilon = 10^{-3}$.

Properties of the False-Position method **خواص الطريقة**

1- هذه الطريقة تتقارب اسرع من طريقة التنصيف (Bisection)

2- نلاحظ عدم استخدام مقياس التوقف $|b_i - a_i| < \varepsilon$ بسبب اذا كانت $b_i - a_i$ تقترب من الصفر فان المقياس الثاني $|f(c_i)| \neq 0$.

3- مضمونة الوصول الى الجذر ولهذا فهي ذات نوع اقترابي شمولي(Global convergent).

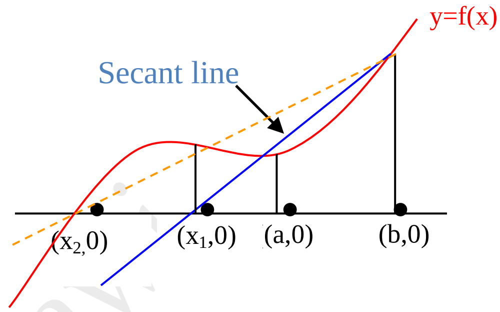
4- سرعة اقتراب الطريقة خطي (Linear) .

(3) Secant Method:

Suppose a continuous function f defined on the interval [a,b] is given, but now we do not force the iterates to bracket a root. The graph of the function f is approximated by a secant line, we get

$$\frac{f(b)-f(a)}{b-a}=\frac{f(b)-y}{b-x} \Rightarrow \frac{f(b)-f(a)}{b-a}=\frac{f(b)-0}{b-x_1}$$

$$\Rightarrow x_1 = b-\frac{f(b)(b-a)}{f(b)-f(a)}=\frac{af(b)-bf(a)}{f(b)-f(a)}$$



Similarly $\quad x_2=\frac{bf(x_1)-x_1 f(b)}{f(x_1)-f(b)}$

In general $\quad x_{i+1}=\frac{x_{i-1}f(x_i)-x_i f(x_{i-1})}{f(x_i)-f(x_{i-1})}$; i=1, …

x_{i+2} is an approximate root if $|x_{i+2}-x_{i+1}|< \varepsilon$ for any i.

<u>**Example (8)**</u>:

Find an approximate root of the equation $x^3-3x+2=0$ by using Secant method and take x_0=-2.6 , x_1=-2.4 , ε=5*10^{-5}.

<u>**Solution**</u>: we use the formula $x_{i+1}=\frac{x_{i-1}f(x_i)-x_i f(x_{i-1})}{f(x_i)-f(x_{i-1})}$; i=1,2, …

i=1 $\Rightarrow$ $$x_2=\frac{x_0 f(x_1)-x_1 f(x_0)}{f(x_1)-f(x_0)}=\frac{(-2.6)f(-2.4)-(-2.4)f(-2.6)}{f(-2.4)-f(-2.6)}$$

$$=\frac{(-2.6)((-2.4)^3-3(-2.4)+2)-(-2.4)((-2.6)^3-3(-2.6)+2)}{((-2.4)^3-3(-2.4)+2)-((-2.6)^3-3(-2.6)+2)}=-2.106599$$

$$|x_2-x_1|=|-2.106599-(-2.4)|>\varepsilon=5*10^{-5}$$

$x_0= x_1=-2.4$ and $f(x_0)=f(x_1)$

$x_1= x_2=-2.106599$ and $f(x_1)=f(x_2)$

i=2 $\Rightarrow$ $$x_3=\frac{x_1 f(x_2)-x_2 f(x_1)}{f(x_2)-f(x_1)}=\frac{(-2.4)f(-2.106599)-(-2.106599)f(-2.4)}{f(-2.106599)-f(-2.4)}$$
$$=-2.0226414$$

$$|x_3-x_2|=|-2.0226414-(-2.106599)|>\varepsilon=5*10^{-5}$$

$x_1= x_2=-2.106599$ and $f(x_1)=f(x_2)$

$x_2= x_3=-2.0226414$ and $f(x_2)=f(x_3)$

i=3 $\Rightarrow$ $x_4=-2.0015111$

$$|x_4-x_3|>\varepsilon=5*10^{-5}$$

$x_2= x_3=-2.0226414$ and $f(x_2)=f(x_3)$

$x_3= x_4=-2.0015111$ and $f(x_3)=f(x_4)$

i=4 $\Rightarrow$ $x_5=-2.0000225$

$$|x_5-x_4|>\varepsilon=5*10^{-5}$$

$x_3= x_4=-2.0015111$ and $f(x_3)=f(x_4)$

$x_4= x_5=-2.0000225$ and $f(x_4)=f(x_5)$

i=5 $\Rightarrow$ $x_6=-2$

$$|x_6-x_5|=0.0000225<\varepsilon=5*10^{-5}$$

Stop iteration with the root $x_6=-2$.

Note:

The method is not guarantee to converge, but it does so with a greater speed than the previous methods when it does converge.

Algorithm (Secant method)

Input: x_0, x_1, ε, f(x)

Step(1): Set i=1 , compute $y_0=f(x_0)$, $y_1=f(x_1)$

Step(2): Evaluate $x = \dfrac{x_0 y_1 - x_1 y_0}{y_1 - y_0}$

Step(3): If $|x - x_1| < \varepsilon$ then print x is a root and stop.

Step(4): else if set i=i+1

Step(5): Set $x_0= x_1$ and $y_0= y_1$, $x_1= x$ and $y_1= f(x)$ then go to step(2)

<u>Home work:</u>

Find an approximate root of equation f(x)=x-0.2sin(x)-0.5=0 in the interval [0.5,1] by using Secant method with $\varepsilon = \dfrac{10^{-4}}{2}$.

Answer : The root is x_7=0.615470301 .

Properties of the Secant method **خواص الطريقة**

1- سرعة الاقتراب في طريقة القاطع فوق الخطية (Supper Linear)

2- تحتاج الى حساب قيمة الدالة مرة واحدة لكل تكرار .

3- نوع الاقتراب محلي (Locally convergent).

4- صيغة الخطأ في طريقة القاطع هي $e_{i+1} = -\dfrac{e_i e_{i-1}}{2} \dfrac{f''(\eta_i)}{f'(\xi_i)}$ حيث كل من ξ_i , η_i تقع في الفترة التي تحتوي على الجذر الحقيقي والقيم x_i, x_{i-1} .