$x_0=0$, $f(x_0)=f(0)=1$

$x_1=0+1=1$, $f(x_1)=f(1)=2$

$$I[f]=\int_0^1 (x^3+1)dx=\frac{h}{2}[f_0+f_1]$$

$$=\frac{1}{2}\,[1+2]=1.5$$

H.W.: n=4

Evaluation the number of subintervals in Trapezoidal rule:

From the error formula

$$E_T=-\frac{(b-a)^3}{12n^2}f''(\theta) \quad , \quad \theta\epsilon(a,b)$$

Let $|f''(\theta)|\le M$ then the error E_T for the composite Trapezoidal rule is less than accuracy ϵ

$$|E_T|=\left|-\frac{(b-a)^3}{12n^2}f''(\theta)\right|\le\epsilon$$

$$\Longrightarrow \quad \frac{(b-a)^3}{12n^2}M\le\epsilon$$

$$\Longrightarrow \quad n\ge\sqrt{\frac{(b-a)^3}{12\epsilon}M}$$

Example(3): Find the value of the integral with accuracy $\epsilon = 0.001$ by using Trapezoidal rule

$$I(f) = \int_{0.5}^{1} \cos x \, dx$$

Solution:

a=0.5 , b=1 , $f(x) = \cos x$ and $\epsilon = 0.001$

f'(x)=-sin x and f''(x)=-cos x $\Longrightarrow |f''(x)| \le 1$ $\Longrightarrow M=1$

$$n \ge \sqrt{\frac{(b-a)^3}{12\,\epsilon} M} = \sqrt{\frac{(1-0.5)^3}{12 * 0.001}(1)} \cong 3.2275$$

$\Longrightarrow n = 4$

$\Longrightarrow h = \frac{b-a}{n} = \frac{1-0.5}{4} = 0.125$

$x_i = x_{i-1} + h$

x	0.5	0.625	0.75	0.875	1
f(x)	0.8776	0.811	0.7317	0.641	0.5403

$$I[f] = \int_{0.5}^{1} \cos x \, dx = \frac{h}{2}[f_0 + 2(f_1 + f_2 + f_3) + f_4]$$

$$\cong 0.3616$$

Algorithm of Trapezoidal rule:

Input: a , b , n , f(x)

Step(1): Evaluate h=(b-a)/n

Step(2): for i=0,1,2,…,n

set x_i=a+ih and $y_i=f(x_i)$

Step(3): set sum=0

Step(4): for i=1,2,…,n-1

find sum =sum + y_i

Step(5): set I=h*[sum+1/2(y_0+y_n)]

Step(6): print I and stop.