(ج) when $\Sigma, \mu$ unknown:

$$L(\underline{\mu}, \Sigma) = (2\pi)^{-\frac{np}{2}} |\Sigma|^{-\frac{n}{2}} e^{-\frac{1}{2}\sum_{i=1}^{n}(\underline{x}_i-\underline{\mu})'\Sigma^{-1}(\underline{x}_i-\underline{\mu})}$$

$$\ln L(\underline{\mu}, \Sigma) = -\frac{np}{2}\ln(2\pi) - \frac{n}{2}|\Sigma| - \frac{1}{2}\sum_{i=1}^{n}(\underline{x}_i-\underline{\mu})'\Sigma^{-1}(\underline{x}_i-\underline{\mu})$$

$$= -\frac{np}{2}\ln(2\pi) - \frac{n}{2}|\Sigma| - \frac{1}{2}tr[\Sigma^{-1}\sum(\underline{x}_i-\underline{\mu})'(\underline{x}_i-\underline{\mu})] \quad \text{----- (1)}$$

باشتقاق المعادلة (1) اشتقاقا جزئيا نسبة الى $(\Sigma, \mu)$ نجد ان

$$\frac{\partial \ln L(\underline{\mu}, \Sigma)}{\partial \mu} = \Sigma^{-1}\sum_{i=1}^{n}(\underline{x}_i-\underline{\mu}) = \underline{0}$$

$$\Rightarrow \hat{\mu} = \underline{\bar{X}} = \begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \vdots \\ \bar{X}_p \end{bmatrix}$$

$$\frac{\partial \ln L(\underline{\mu}, \Sigma)}{\partial \Sigma} = -\frac{n}{2}\Sigma^{-1} + \frac{1}{2}\Sigma^{-1}\sum_{i=1}^{n}(\underline{x}_i-\underline{\mu})(\underline{x}_i-\underline{\mu})'\Sigma^{-1}$$

$$= \underline{0}$$

$$\therefore \hat{\Sigma} = \frac{1}{n}C$$

حيث ان

$$C = \sum_{i=1}^{n}(\underline{x}_i-\bar{\underline{x}})(\underline{x}_i-\bar{\underline{x}})'$$

اي ان :

$$C = \begin{bmatrix} \sum_{i=1}^{n}(x_{1i}-\bar{x}_1)^2 & \sum_{i=1}^{n}(x_{1i}-\bar{x}_1)(x_{2i}-\bar{x}_2) & \cdots & \sum_{i=1}^{n}(x_{1i}-\bar{x}_1)(x_{pi}-\bar{x}_p) \\ & \sum_{i=1}^{n}(x_{2i}-\bar{x}_2)^2 & & \\ & & \ddots & \\ & & & \sum(x_{pi}-\bar{x}_p)^2 \end{bmatrix}$$

ملاحظة :

ان المقدر غير المتحيز لمصفوفة $(\Sigma)$ هو عندما $(\mu)$ غير معلومة

كون اننا قدرنا $(\mu)$ لذلك نطرح $(1)$ من درجات الحرية

$$\Rightarrow \hat{\Sigma}_{unb} = \frac{C}{n-1}$$

اما المقدر $\hat{\Sigma} = \frac{C}{n}$ هو مقدر متحيز

اي ان :

$$\hat{\Sigma} = \frac{C}{n}$$

عندما $\mu$ غير معلومة

where $C = \sum(x_i-\bar{x})(x_i-\bar{x})'$

$$E\hat{\Sigma} = \frac{1}{n}EC = \frac{n-1}{n}\Sigma$$

$$\therefore E\hat{\Sigma} \neq \Sigma$$

$\therefore$ هو مقدر متحيز

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## Remarki

يمكن كتابة مصفوفة C بالشكل الآتي:

$$C = \sum_{i=1}^{n} (X_i - \bar{X})(X_i - \bar{X})' = \sum_{i=1}^{n} x_i x_i' - n \bar{X} \bar{X}'$$

$$= \begin{bmatrix} \Sigma x_{1i}^2 & \Sigma x_{1i} x_{2i} & \cdots & \Sigma x_{1i} x_{pi} \\ & \Sigma x_{2i}^2 & \Sigma x_{2i} x_{3i} \cdots & \Sigma x_{2i} x_{pi} \\ & & \ddots & \\ & & & \Sigma x_{pi}^2 \end{bmatrix} - n \begin{bmatrix} \bar{x}_1 & \bar{x}_1 \bar{x}_2 & \cdots & \bar{x}_1 \bar{x}_p \\ & \bar{x}_2 & \bar{x}_2 \bar{x}_3 \cdots & \bar{x}_2 \bar{x}_p \\ & & \ddots & \\ & & & \bar{x}_p^2 \end{bmatrix}$$