مسائل البرمجة الخطية التي يرمز لها بالرمزLpp

$Max\ (Min)\ z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$ ....(1) (دالة الهدف)

Subject to:-

$$\left.\begin{array}{l} a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n = b_1 \\ \quad . \qquad . \qquad\quad . \quad . \\ \quad . \qquad . \qquad\quad . \quad . \\ a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n = b_m \end{array}\right\}$$ ....(2) (القيود)

$x_1 \ge 0\ ,\ x_2 \ge 0\ , \dots ,\ x_n \ge 0$ ....(3) (الشروط اللاسالبية)

$Max\ (Min)\ z = \sum_{j=1}^{n} c_j x_j$ ....(1) (دالة الهدف)

Subject to:-

$\sum_{j=1}^{n} a_{ij}\ x_j = b_i\ ,\ i = 1,2,\dots,m$ ....(2) (القيود)

$x_j \ge 0 \quad \forall\ j = 1,2,\dots,n$ ....(3) (الشروط اللاسالبية)

$Max\ (Min)\ z = c^T x$ ....(1) (دالة الهدف)

Subject to:-

$A x = B$ ....(2) (القيود)

$x \ge 0$ ....(3) (الشروط اللاسالبية)

حيث أن $A = [m \times n]$ مصفوفة ، $x = (x_1, x_2, \dots, x_n)^T,\ \ c = (c_1, c_2, \dots, c_n)^T,\ B = (b_1, b_2, \dots, b_m)^T$.

A linear programming problem may be defined as the problem of ***maximizing*** or ***minimizing*** a linear function subject to linear constraints. The constraints may be equalities or inequalities. All variables are nonnegative.

Here is a simple example. Find numbers x_1 and x_2 that maximize the sum $x_1 + x_2$ subject to the constraints $x_1 \ge 0$, $x_2 \ge 0$ and

$$\begin{aligned} S.t.: \quad x_1 + 2x_2 &\le 4 \\ 4x_1 + 2x_2 &\le 12 \\ -x_1 + x_2 &\le 1 \end{aligned} \qquad \dots\dots\dots\dots (Q)$$

In this problem there are two unknowns, and five constraints. All the constraints are inequalities and they are all linear in the sense that each involves an inequality in some linear function of the variables. The first two constraints, $x_1 \geq 0$ and $x_2 \geq 0$, are special.

These are called ***non-negativity constraints*** and are often found in linear programming problems. The other constraints are then called the ***main constraints***. The function to be maximized (or minimized) is called the ***objective function***. Here, the objective function is $x_1 + x_2$.

It is easy to see in general that the objective function, being linear, always takes on its maximum (or minimum) value at a corner point of the constraint set, provided the constraint set is bounded. Occasionally, the maximum occurs along an entire edge or face of the constraint set, but then the maximum occurs at a corner point as well.

Not all linear programming problems are so easily solved. There may be many variables and many constraints. Some variables may be constrained to be nonnegative and others unconstrained. Some of the main constraints may be equalities and others inequalities. However, two classes of problems, called here the ***standard maximum problem*** and the ***standard minimum problem***, play a special role. In these problems, all variables are constrained to be nonnegative, and all main constraints are inequalities.

All Linear Programming Problems Can be Converted to Standard Form.

A linear programming problem was defined as maximizing or minimizing a linear function subject to linear constraints. All such problems can be converted into the form of a standard maximum problem by the following techniques. A minimum problem can be changed to a maximum problem by multiplying the objective function by -1. Similarly, constraints of the form $\sum_{j=1}^{n} a_{ij} x_j \geq b_i$, $i = 1, 2, 3, \ldots, m$ can be changed into the form $\sum_{j=1}^{n} - a_{ij} x_j \leq - b_i$, $i = 1, 2, 3, \ldots, m$.

Two other problems arise.

(1) Some constraints may be equalities. An equality constraint $\sum_{j=1}^{n} a_{ij} x_j = b_i$, $i = 1, 2, 3, \ldots, m$ may be removed, by solving this constraint for some x_j for which $a_{ij} \neq 0$ and substituting this solution into the other constraints and into the objective function wherever x_j appears. This removes one constraint and one variable from the problem.

(2) Some variable may not be restricted to be nonnegative. An unrestricted variable, x_j, may be replaced by the difference of two nonnegative variables, xj = uj − vj, where uj ≥ 0 and vj ≥ 0. This adds one variable and two non-negativity constraints to the problem.

Among the conditions of this formula are :

1. The factor objective should be of type Max or Min.

2. The constraints should be written on an inequality with the sign (≤) or (≥) or equality (=).

3.The variables should be of type Restricted variable or unrestricted in sign.

$$
\begin{aligned}
&Minimum or\ Maximum Z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n \\
&Subject to \\
&\quad a_{11} x_1 + a_{12} x_2 + \ldots + a_{1n} x_n \leq or = or \geq b_1 \\
&\quad a_{21} x_1 + a_{22} x_2 + \ldots + a_{2n} x_n \leq or = or \geq b_2 \\
&\quad \vdots \\
&\quad a_{m1} x_1 + a_{m2} x_2 + \ldots + a_{mn} x_n \leq or = or \geq b_m \\
&\quad x_1 \ , \ x_2 \ , \ \ldots\ldots \ , \ x_n \geq 0
\end{aligned}
$$

Conical form :

Among the conditions of this formula are :

1. The factor objective should be of type Max only.
2. The coefficient should be written on an inequality with the sign ($\leq$).
3. The variables should be of type Restricted variable.

All Linear Programming Problems Can be Converted to Standard Form.

A linear programming problem was defined as maximizing or minimizing a linear function subject to linear constraints. All such problems can be converted into the form of a standard maximum problem by the following techniques. A minimum problem can be changed to a maximum problem by multiplying the objective function by −1. Similarly, constraints of the form $\sum_{j=1}^{n} a_{ij} x_j \geq b_i \ , \ i = 1, 2, 3, \ldots, m$ can be changed into the form $\sum_{j=1}^{n} - a_{ij} x_j \leq - b_i \ , \ i = 1, 2, 3, \ldots, m$.

Two other problems arise.

(1) Some constraints may be equalities. An equality constraint $\sum_{j=1}^{n} a_{ij} x_j = b_i \ , \ i = 1, 2, 3, \ldots, m$ may be removed, by solving this constraint for some x_j for which $a_{ij} \neq 0$ and substituting this solution into the other constraints and into the objective function wherever x_j appears. This removes one constraint and one variable from the problem.

(2) Some variable may not be restricted to be nonnegative. An unrestricted variable, x_j, may be replaced by the difference of two nonnegative variables, xj = uj − vj, where uj $\geq$ 0 and vj $\geq$ 0. This adds one variable and two non-negativity constraints to the problem.

The Treatments :

1. lf variables unrestricted in sign. For example Y.

$$Y = L - M \qquad L\,,\, M \geq 0$$

Let

$$L = 0 \;,\; M = 6 \qquad , \qquad Y = 0 - 6 = -6$$

or

$$L = 10 \;,\; M = 0 \qquad , \qquad Y = 10 - 0 = 10$$

or

$$L = 4 \;,\; M = 2 \qquad , \qquad Y = 4 - 2 = 2$$

2. If objective function should be of type Min $\Rightarrow$ Max

$$Min\ Z = c_1x_1 + c_2x_2 + ... + c_nx_n \quad \Rightarrow \quad Max Z = -c_1x_1 - c_2x_2 - ... - c_nx_n$$

3. If

$$|a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n| \le b_1 \quad \Rightarrow \quad \begin{array}{l} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \le b_1 \\ a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \ge -b_1 \end{array}$$

1. If

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \ge b_1 \quad \Rightarrow \quad -a_{11}x_1 - a_{12}x_2 - ... - a_{1n}x_n \le -b_1$$

2. If

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \quad \Rightarrow \quad \begin{array}{l} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \le b_1 \\ a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \ge b_{11} \end{array}$$

Note :

The linear programming should contain a number of written constrains in the form of equations. If the sign of the inequality is less or equal to, then the slack variable is added and it is symbolized as S. However, if the sign of the inequality is greater or equal to then the variable S from the left side is reduced and as follows :

1. If the relation is less or equal to ($\le$) .

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \le b_1 \quad \Rightarrow \quad a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n + S_1 = b$$

2. If the relation is greater or equal to ($\ge$) .

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \ge b_1 \quad \Rightarrow \quad a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n - S_1 = b$$