تمارين إضافية لطرائق الحذف الفترات

Fibonacci Method

Fibonacci Method: Fibonacci Method can be used to find minimum of function one variable. This method markers use of the sequence if Fibonacci number $[f_n]$.

These are defined as:

$f_\circ = f_1 = 1$

$f_n = f_{n-1} + f_{n-2} \quad , \quad n = 2,3,4,5 \dots.$

$f_2 = f_1 + f_2$

$= 1 + 1 = 2$

$f_3 = f_2 + f_3$

$= 2 + 1 = 3$

n	f_n
0	1
1	1
2	2
3	3
4	5
5	8
6	13
7	21
8	34

The basic steps of this method are:

$Step\ (1)$: $Input\ [\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{\epsilon}, \boldsymbol{n}, \boldsymbol{f}]\ is\ number\ of\ Fibonacci.$

$Step\ (2)$: $Define\ \boldsymbol{D} = \frac{\boldsymbol{f_{n-1}}}{\boldsymbol{f_n}}(\boldsymbol{b} - \boldsymbol{a}).$

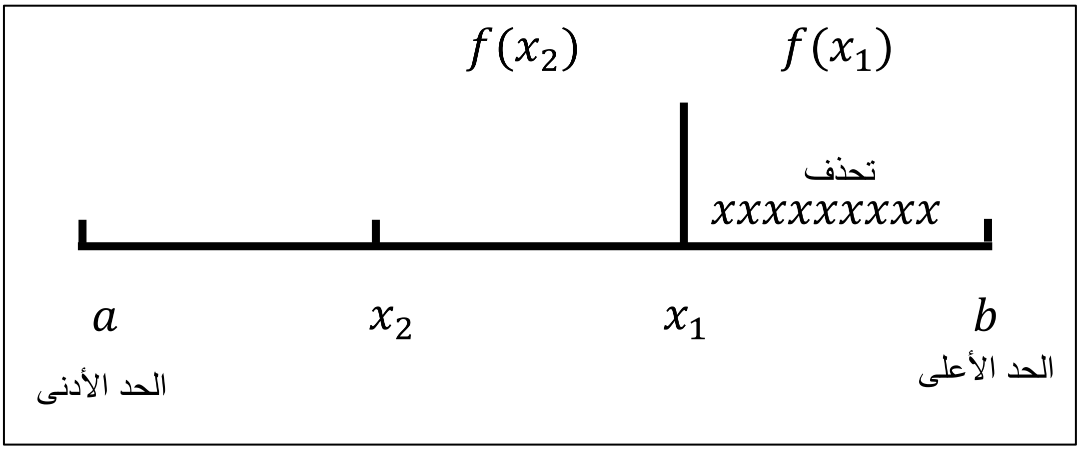
$Step\ (3)$: $Compute\ \boldsymbol{x_1} = \boldsymbol{a} + \boldsymbol{D}\ and\ \boldsymbol{x_2} = \boldsymbol{b} - \boldsymbol{D}$

$Step\ (4)$: $Evaluate\ the\ function\ \boldsymbol{f}(\boldsymbol{x})\ at\ (\boldsymbol{x_1})\ and\ (\boldsymbol{x_2})\ to\ obtain\ \boldsymbol{f}(\boldsymbol{x_1})\ and\ \boldsymbol{f}(\boldsymbol{x_2}).$

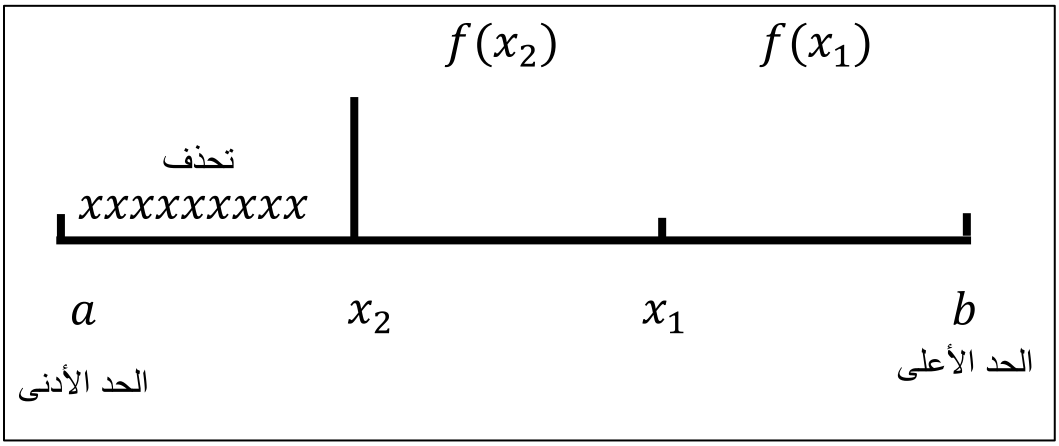
$Step$ (5):

a) If $x_1 > x_2$

1) If $f(x_1) > f(x_2)$ then deleted the interval $[x_1, b]$ and new interval $[a, x_1]$:

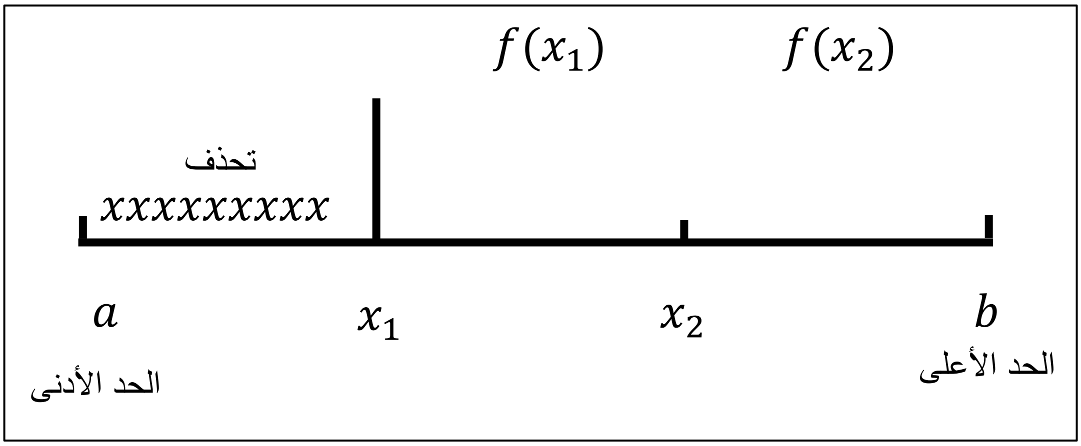


2) If $f(x_1) < f(x_2)$ then deleted the interval $[a, x_2]$ and new interval $[x_2, b]$:

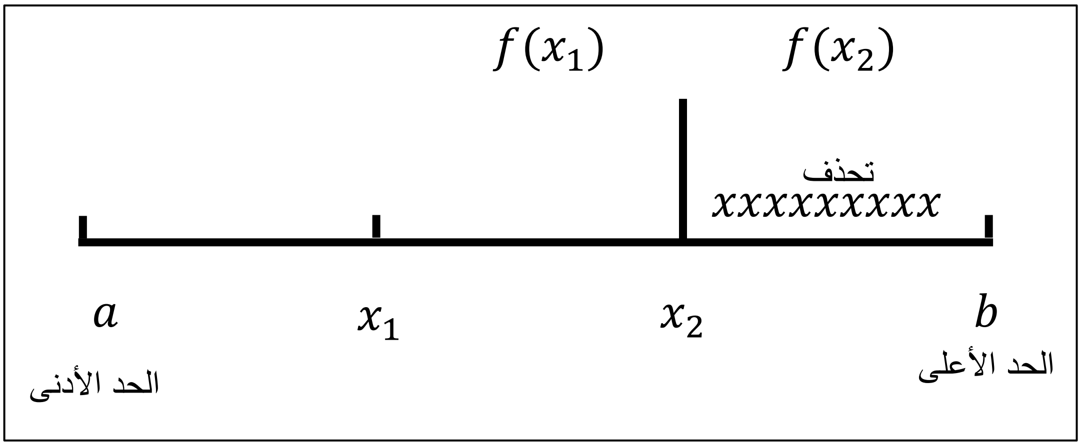


b) If $x_1 < x_2$

1) If $f(x_1) > f(x_2)$ then deleted the interval $[a, x_1]$ and new interval $[x_1, b]$:



2) If $\boldsymbol{f(x_1) < f(x_2)}$ then deleted the interval $[\boldsymbol{x_2, b}]$ and new interval $[\boldsymbol{a, x_2}]$:



$Step\ (6): If\ |\boldsymbol{b - a}| < \epsilon\ Then\ \left[\boldsymbol{R} = \frac{\boldsymbol{a+b}}{\boldsymbol{2}}\right]\ and\ stop, otherwise\ go\ to\ step\ (2).$

Example: use Fibonacci method to find minimum $\left[\boldsymbol{f(x)} = \frac{\boldsymbol{3-4x}}{\boldsymbol{1+x^2}}\right]$

$\left[\overset{a}{1} < x < \overset{b}{6.5}\right]$

$\boldsymbol{n = 8, \varepsilon = 0.05}$

Solution:

$$\boldsymbol{D} = \frac{f_{n-1}}{f_n}(b-a)$$

$$= \frac{f_7}{f_8}(6.5 - 1) \rightarrow \frac{21}{34}(5.5) = \mathbf{3.4}$$

$$\boldsymbol{x_1} = a + D$$

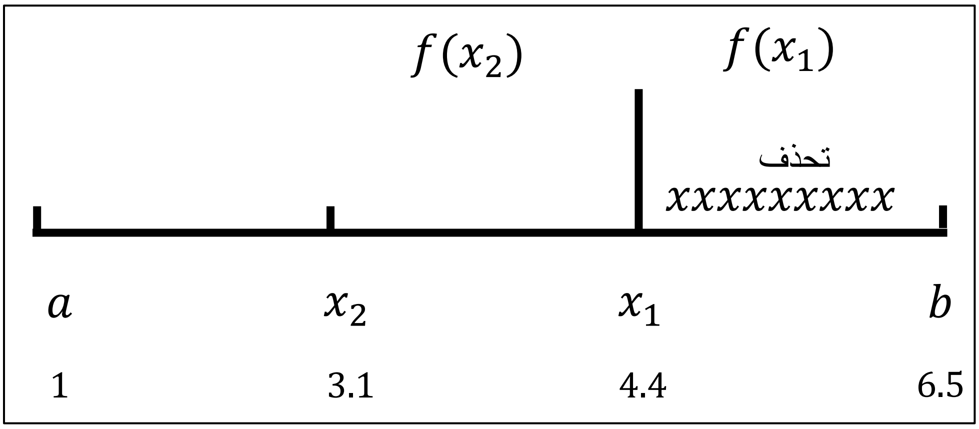
$$= 1 + 3.4 = \mathbf{4.4}$$

$$\boldsymbol{x_2} = b - D$$

$$= 6.5 - 3.4 = \mathbf{3.1}$$

$$\boldsymbol{f(x_1)} = \frac{3 - 4(4.4)}{1 + (4.4)^2} = \mathbf{-0.71}$$

$$\boldsymbol{f(x_2)} = \frac{3 - 4(3.1)}{1 + (3.1)^2} = \mathbf{-0.89}$$



The new interval is:

$[a, x_1] = [1, 4.4]$

$|b - a| < \epsilon$

$|4.4 - 1| = \mathbf{3.4 > \epsilon}$ نستمر بالحل

Iteration (2):

$$\boldsymbol{D} = \frac{f_6}{f_7}(b - a)$$

$$= \frac{13}{21}(4.4 - 1) = \mathbf{2.1}$$

$$\boldsymbol{x_1} = a + D$$

$$= 1 + 2.1 = \mathbf{3.1}$$

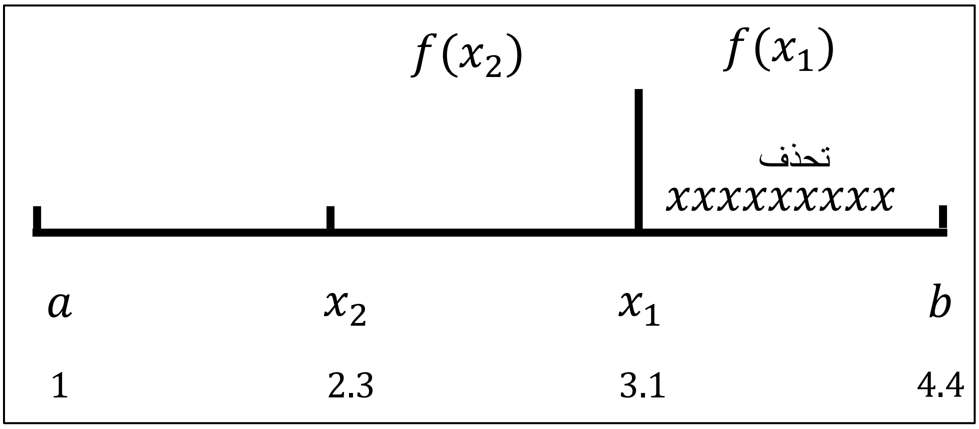
$$\boldsymbol{x_2} = b - D$$

$$= 4.4 - 2.1 = \mathbf{2.3}$$

$$\boldsymbol{f(x_1)} = \frac{3 - 4(3.1)}{1 + (3.1)^2} = \mathbf{0.89}$$

$$\boldsymbol{f(x_2)} = \frac{3 - 4(2.3)}{1 + (2.3)^2} = \mathbf{-0.99}$$

الرسم في الصفحة القادمة



The new interval is

$[a, x_1] = [1, 3.1]$

$|b - a| = \mathbf{2.1 > \epsilon}$ نستمر بالحل

Iteration (3):

$$\boldsymbol{D} = \frac{f_5}{f_6} = (b - a)$$

$$= \frac{8}{13}(2.1) = \mathbf{1.29}$$ نستمر بالحل

Golden Section Method

$$Golden\ Ratio \Longrightarrow \boldsymbol{T} = \frac{\mathbf{1}+\sqrt{\mathbf{5}}}{\mathbf{2}} = \mathbf{1.618}$$

$The\ basic\ steps\ of\ this\ method\ are$:

$Step\ (1)$: $Set\ the\ initial\ interval\ (\boldsymbol{a}, \boldsymbol{b})\ and\ \boldsymbol{T} = \frac{\mathbf{1}+\sqrt{\mathbf{5}}}{\mathbf{2}} = \mathbf{1.618}\ ,\ \boldsymbol{\epsilon}$

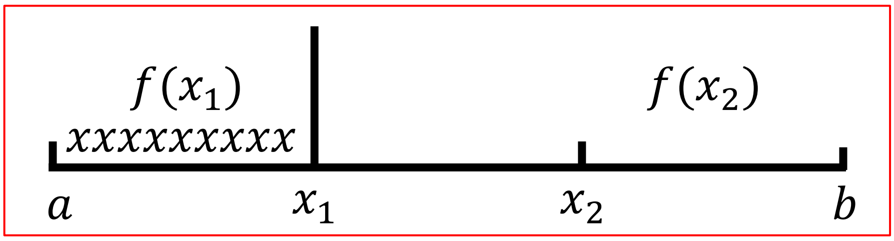
$Step\ (2)$: $Compute\ \left[\boldsymbol{x_1} = \frac{\boldsymbol{T}-\mathbf{1}}{\boldsymbol{T}}(\boldsymbol{b}-\boldsymbol{a}) + \boldsymbol{a}\right]\ and\ \left[\boldsymbol{x_2} = \frac{\mathbf{1}}{\boldsymbol{T}}(\boldsymbol{b}-\boldsymbol{a}) + \boldsymbol{a}\right]$

Step (3): Find the function values of $(\boldsymbol{x_1})$ and $(\boldsymbol{x_2})$ and $\boldsymbol{f}(\boldsymbol{x_1})\ and\ \boldsymbol{f}(\boldsymbol{x_2})$

$Step\ (4)$:

a)

$If\ \boldsymbol{f}(\boldsymbol{x_1}) > \boldsymbol{f}(\boldsymbol{x_2}).\ Then\ deleted\ the\ interval\ (\boldsymbol{a}, \boldsymbol{x_1})\ and\ the\ new\ interval\ (\boldsymbol{x_1}, \boldsymbol{b})$.



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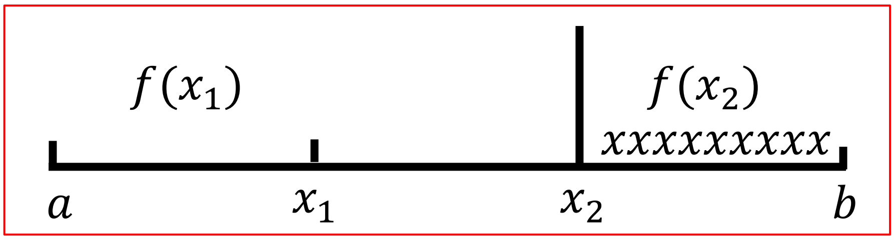
b)

$If\ \boldsymbol{f}(\boldsymbol{x_1}) < \boldsymbol{f}(\boldsymbol{x_2}).\ Then\ deleted\ the\ interval\ (\boldsymbol{x_2}, \boldsymbol{b})\ and\ the\ new\ interval\ (\boldsymbol{a}, \boldsymbol{x_2})$.



$Step$ (5): $check$ $|b-a| < \epsilon$, $R = \frac{a+b}{2}$. $Otherwise$ and go $step$ (2).

The golden ratio flowchart

Example: find the minimum $\boldsymbol{f(x) = (x-4)^2}$, $\boldsymbol{2 < x < 5}$ $\boldsymbol{or\ x \in [2, 5]}$, $\boldsymbol{\epsilon = 0.05}$

Solution:

$\boldsymbol{Iteration\ (1)}$:

$$x_1 = \frac{T-1}{T}(b-a) + a$$

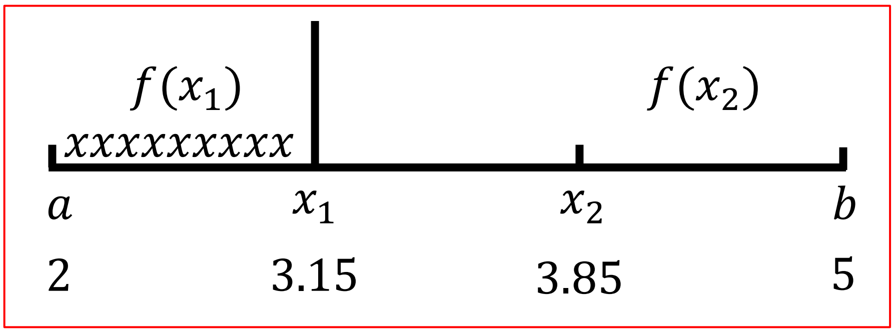
$$\boldsymbol{x_1} = \frac{1.618-1}{1.618}(5-2) + 2 \Longrightarrow \frac{0.618}{1.618}(3) + 2 = \boldsymbol{3.15}$$

$$x_2 = \frac{1}{T}(b-a) + a$$

$$\boldsymbol{x_2} = \frac{1}{1.618}(5-2) + 2 \Longrightarrow \frac{1}{1.618}(3) + 2 = \boldsymbol{3.85}$$

$$\boldsymbol{f(x_1)} = (3.15-4)^2 \Longrightarrow = \boldsymbol{0.7225}$$

$$\boldsymbol{f(x_2)} = (3.85-4)^2 \Longrightarrow = \boldsymbol{0.0225}$$



$\therefore The\ new\ interval: (x_1, b) = (3.15, 5)$

$\boldsymbol{|b-a|} = |5-3.15| = \boldsymbol{1.85} > \boldsymbol{\epsilon}$ نستمر بالحل

$\boldsymbol{Iteration\ (2)}$:

$$x_1 = \frac{T-1}{T}(b-a) + a$$

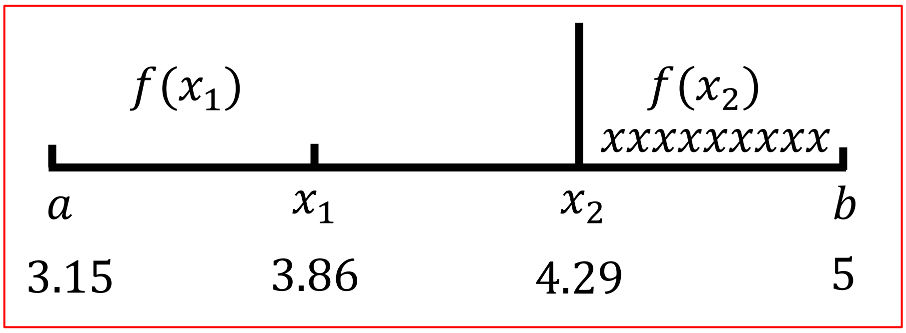
$$\boldsymbol{x_1} = \frac{1.618-1}{1.618}(5-3.15) + 3.15 \Longrightarrow \frac{0.618}{1.618}(1.85) + 3.15 = \boldsymbol{3.86}$$

$$x_2 = \frac{1}{T}(b-a) + a$$

$$\boldsymbol{x_2} = \frac{1}{1.618}(5-3.15) + 3.15 \Longrightarrow \frac{1}{1.618}(1.85) + 3.15 = \mathbf{4.29}$$

$$\boldsymbol{f(x_1)} = (3.86-4)^2 \Longrightarrow = \mathbf{0.0196}$$

$$\boldsymbol{f(x_2)} = (4.29-4)^2 \Longrightarrow = \mathbf{0.0841}$$

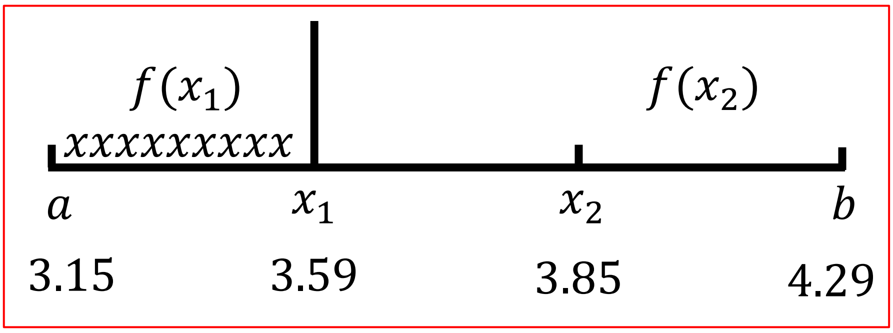


$\therefore$ *The new interval*: $(a, x_2) = (3.15, 4.29)$

$\boldsymbol{|b-a|} = |4.29 - 3.15| = \mathbf{1.14} > \boldsymbol{\epsilon}$ نستمر بالحل

***Iteration* (3):**

$x_1 = \mathbf{3.59}$, $x_2 = \mathbf{3.85}$, $f(x_1) = \mathbf{0.1681}$, $f(x_2) = \mathbf{0.0225}$

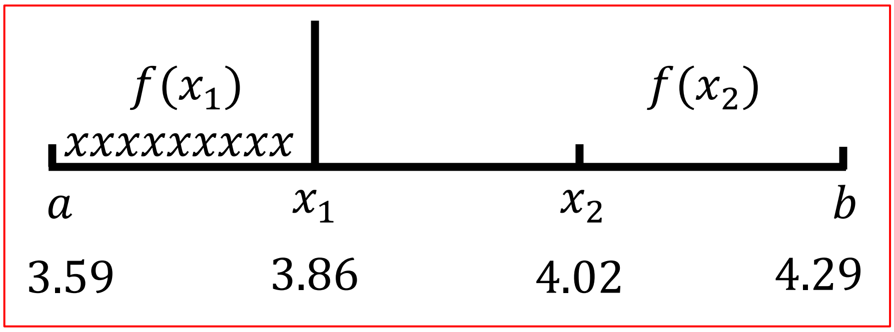


$\therefore$ *The new interval*: $(x_1, b) = (3.59, 4.29)$

$\boldsymbol{|b-a|} = |4.29 - 3.59| = \mathbf{0.7} > \boldsymbol{\epsilon}$ نستمر بالحل

***Iteration* (4):**

$x_1 = \mathbf{3.86}$, $x_2 = \mathbf{4.02}$, $f(x_1) = \mathbf{0.0196}$, $f(x_2) = \mathbf{0.0004}$



$$(x_1, b) = (3.86, 4.29), \qquad |\boldsymbol{b - a}| = |4.29 - 3.86| = \mathbf{0.43} < \boldsymbol{\epsilon =} 0.5$$

$$\boldsymbol{R} = \frac{a+b}{2} = \frac{3.86 + 4.29}{2} = \mathbf{4.075}$$