Partial Derivatives One Variable at a Time,

المشتقات الجزئية متغير واحد في كل مرة

Definitions and Theorems: **تعاريف ونظريات**

- Let $z = f(x,y)$ is a function consisting of two variables (x,y) then the **partial derivative** means that we can derive in terms of (x) as the variable and (y) is a constant value and we can derive in terms of (y) as the variable and (x) is a constant value, and it can be expressed in the following form:

لتكن الدالة z المكونة من متغيرين(x,y) فان المشتقة الجزئية يقصد بها بانه ممكن ان نشتق بدلالة (x) باعتباره متغيراً و (y) قيمة ثابتة وممكن ان نشتق بدلالة (y) باعتباره متغيراً و (x) قيمة ثابتة ، ويمكن التعبير عنها بالشكل الاتي:

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} = f_x \quad , \quad \frac{\partial z}{\partial y} = \frac{\partial f}{\partial y} = f_y$$

- Higher-order partial derivatives:

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = f_{xx} \quad , \quad \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = f_{yy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x} = f_{xy} \quad , \quad \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y} = f_{yx}$$

- Laplace's partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$. A function that satisfies this equation is said to be **harmonic**.

معادلة لابلاس التفاضلية الجزئية : يقال للدالة التي تحقق معادلة لابلاس بانها توافقية

Example (1): Find the first partial differential of the function $z = f(x,y) = x^3 + y^4 + \sin(xy)$

Solve:

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} = 3x^2 + y\cos(xy)$$

$$\frac{\partial z}{\partial y} = \frac{\partial f}{\partial y} = 4y^3 + x\cos(xy)$$

Example (2): Find $f_x, f_y, f_{xy}, f_{yx}, f_{xx}$ & f_{yy} of the function $f(x,y) = x^3 + 3y^2x + y^4$

Solve:

$$f_x = \frac{\partial f}{\partial x} = 3x^2 + 3y^2$$

$$f_y = \frac{\partial f}{\partial y} = 6yx + 4y^3$$

$$f_{xy} = \frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x} = 6y$$

$$f_{yx} = \frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y} = 6y$$

لاحظ أن المشتقتين الجزئيتين المختلطتين متساويتان. وفي ظل فرضيات مناسبة، يكون هذا صحيحًا دائمًا بالنسبة للمشتقات الجزئية المختلطة

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = 6x$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = 6x + 12y^2$$

Example (3): Find f_x, f_y & f_z of the function $f(x, y, z) = xy + yz^2 + xz$

Solve:

$$f_x = \frac{\partial f}{\partial x} = y + z$$

$$f_y = \frac{\partial f}{\partial y} = x + z^2$$

$$f_z = \frac{\partial f}{\partial z} = 2yz + x$$

Example (4): Find f_x, f_y, f_{xy}, f_{yx}, f_{xx} & f_{yy} of the function $f(x, y) = sin(x) + e^y + xy$

Solve:

$$f_x = \frac{\partial f}{\partial x} = cos(x) + y$$

$$f_y = \frac{\partial f}{\partial y} = e^y + x$$

$$f_{xy} = \frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x} = 1$$

$$f_{yx} = \frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y} = 1$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = -sin(x)$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = e^y$$

Example (5): Show that $z = \boldsymbol{f(x,y) = e^x sin(y)}$ is a solution to Laplace's equation

Solve:

$$\frac{\partial z}{\partial x} = \boldsymbol{e^x sin(y)} \qquad \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial x^2} = \boldsymbol{e^x sin(y)}$$

$$\frac{\partial z}{\partial y} = \boldsymbol{e^x cos(y)} \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial y^2} = \boldsymbol{-e^x sin(y)}$$

Laplace's partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \boldsymbol{0}$

$$\boldsymbol{e^x sin(y) + \left(-e^x sin(y)\right) = e^x sin(y) - e^x sin(y) = 0}$$

Example (6): for the function $f(x,y) = x^2 - xy + y^2 - 5x + y$ find all value of x and y such that $f_x = 0$ & $f_y = 0$.

Solve:

$$f_x = \frac{\partial f}{\partial x} = 2x - y - 5$$

$0 = 2x - y - 5 \qquad \ldots.(1)$

$$f_y = \frac{\partial f}{\partial y} = -\mathrm{x} + 2\mathrm{y} + 1$$

$0 = -\mathrm{x} + 2\mathrm{y} + 1 \qquad \ldots(2)$

Multiply the Eq.1 by (2)

$4x - 2y - 10 = 0 \qquad \ldots.(1)$

$\underline{-\mathrm{x} + 2\mathrm{y} + 1 = 0 \qquad \ldots.(2)}$ بالجمع

$3x - 9 = 0 \quad \rightarrow \quad 3x = 9 \quad \rightarrow \quad x = 3$

Substitute x in the Eq.1

$0 = 2(3) - y - 5$

$0 = 1 - y \quad \rightarrow \quad y = 1$

H.W:

1- Find $\boldsymbol{f_x}$ & $\boldsymbol{f_y}$ of the function $\boldsymbol{f(x,y) = sin(5x)cos(5y)}$

2- Find $\boldsymbol{f_x}$, $\boldsymbol{f_y}$, $\boldsymbol{f_{xy}}$, $\boldsymbol{f_{yx}}$, $\boldsymbol{f_{xx}}$ & $\boldsymbol{f_{yy}}$ of the function $\boldsymbol{f(x,y) = x^2 - 2xy + 3y^2}$

3- Find $\boldsymbol{f_{xx}}$ & $\boldsymbol{f_{yy}}$ of the function $\boldsymbol{f(x,y) = e^y tan(x)}$