Big – M - method :

The first one involves the occurrence of an artificial variable in the factor objective with a coefficient of (M) if the factor objective is of type Min but if it is of type Max, it will appear with a coefficient of (-M).

When the relation is of type greater or equal to the slack variable are reduced from the left side so as to fulfill the conciliation of the standard formula. After discarding the basic variables from the model ($x_1, x_2, x_3, ..., x_n$) negative signs will appear on the right side and this is in conflict with the feasible solution. In other words, the presence of the negative signs will prevent (or prevents)obtaining the feasible solution which is the base for obtaining the optimal solution. For this important reason, it has been the case to depend on other variable which are called artificial. These are added to the model after reducing the slack variables in order to the obtain the possible solution. Similarly, when the constraints are of type equal to the artificial variables are added .

After obtaining the possible solution, these artificial variables should be removed from the simplex table because its presence during the stage of solving the simplex would impede obtaining the optimal solution.

To illustrate this we prefer to solve the following example :

$$
\begin{aligned}
&MinZ = -3x_1 - 2x_2 - x_3 \\
&S.t. \quad 4x_1 + 3x_2 + 5x_3 \le 10 \\
&\qquad\quad x_1 + x_2 + x_3 = 1
\end{aligned}
$$

Solution:

$$
\begin{gathered}
MinZ = -3x_1 - 2x_2 - x_3 \\
4x_1 + 3x_2 + 5x_3 + \delta_1 = 10 \\
x_1 + x_2 + x_3 = 1
\end{gathered}
$$

…………………………………………..

$$
\begin{gathered}
MinZ = -3x_1 - 2x_2 - x_3 + 100R_1 \\
4x_1 + 3x_2 + 5x_3 + \delta_1 = 10 \\
x_1 + x_2 + x_3 + R_1 = 1
\end{gathered}
$$

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$$
\begin{aligned}
Z^{New} &= -3x_1 - 2x_2 - x_3 + 100R_1 - 100\{2\} \\
&= -3x_1 - 2x_2 - x_3 + 100R_1 - 100x_1 - 100x_2 - 100x_3 + 100 - 100R_1 \\
&= -103x_1 - 102x_2 - 101x_3 + 100
\end{aligned}
$$

Basic	b_i	x_1	x_2	x_3	δ_1	R_1	ch
δ_1	10	4	3	5	0	0	2.5
R_1	1	1	1	1	0	1	1
$-U$	-100	-103	-102	-101	0	0	

Basic	b_i	x_1	x_2	x_3	δ_1	R_1
δ_1	6	0	-1	1	1	-4
x_1	1	1	1	1	0	1
$-U$	3	0	1	2	0	103

Example:

$$
\begin{aligned}
&MinU = 2x_1 + x_2 - x_3 - x_4 \\
&S.t.: \quad x_1 - x_2 + 2x_3 - x_4 = 2 \\
&\quad 2x_1 + x_2 - 3x_3 + x_4 = 6 \\
&\quad x_1 + x_2 + x_3 + x_4 = 7 \\
&\quad \forall x_j \geq 0 \qquad j = 1,2,3,4
\end{aligned}
$$

Solution:

$$
\begin{aligned}
&MinU = 2x_1 + x_2 - x_3 - x_4 + 100x_5 + 100x_6 + 100x_7 \\
&S.t.: \quad x_1 - x_2 + 2x_3 - x_4 + x_5 = 2 \\
&\quad 2x_1 + x_2 - 3x_3 + x_4 + x_6 = 6 \\
&\quad x_1 + x_2 + x_3 + x_4 + x_7 = 7 \\
&\quad \forall x_j \geq 0 \qquad j = 1,\dots,7
\end{aligned}
$$

$$
\begin{aligned}
MinU &= U - 100[4x_1 + x_2 + x_4 + x_5 + x_6 + x_7 - 15] \\
&= 2x_1 + x_2 - x_3 - x_4 + 100x_5 + 100x_6 + 100x_7 - 400x_1 - 100x_2 - 100x_4 - 100x_5 - 100x_6 - 100x_7 + 1500 \\
MinU &= -398x_1 - 99x_2 - x_3 - 101x_4 + 1500
\end{aligned}
$$

Basic	b_i	x_1	x_2	x_3	x_4	x_5	x_6	x_7	$b_{i/2}$
x_5	2	1	-1	2	-1	1	0	0	2
x_6	6	2	1	-3	1	0	1	0	3
x_7	7	1	1	1	1	0	0	1	7
$-u$	-1500	-398	-99	-1	-101	0	0	0	$b_{i/2}$
x_1	2	1	-1	2	-1	1	0	0	-2
x_6	2	0	3	-7	3	-2	1	0	2/3
x_7	5	6	2	-1	2	-1	0	1	5/2
- u	-704	0	-497	795	-499	398	0	0	$b_{i/2}$
x_1	8/3	1	0	-1/3	0	1/3	1/3	0	-8
x_4	2/3	0	1	-7/3	-1	-2/3	1/3	0	5/-7
x_7	11/3	0	0	11/3	0	-7/3	-2/3	1	1
$-u$	-371.3	0	2	-369.3	0	65.3	166.3	0	
x_1	3	1	0	0	0	12/33	9/33	0	
x_4	3	0	1	0	1	-29/33	-3/33	0	
x_3	1	0	0	1	0	1/11	-2/11	1	
$-u$	-2	0	0	0	0	98.27	99.15	0	