. Fibonacci method

The Fibonacci method can be used to find the minimum of a function of one variable even if the function is not continuous. This method makes use of the sequence of Fibonacci numbers, theses numbers are defined as :

$$F_0 = F_1 = 1$$
$$F_n = F_{n-1} + F_{n-2} \quad : \; n = 2, 3, 4, , \ldots$$

which yield the sequence $1, 1, ,2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots.$. The out line of the method is as follows :

1. Set the initial interval $[x_1 \, , \, x_2]$ and find $T = \dfrac{F_{n-2}}{F_n} L_0$.
2. Find the new interval $[x_3 \, , \, x_4]$:

$$x_3 = x_1 + T \qquad \text{and} \qquad x_4 = x_2 - T$$

3. Find the function value of x_3 and x_4, i.e. f_3 and f_4.
4. Check if $f_3 > f_4$ set $x_1 = x_3$ and go to step 5, else set $x_2 = x_4$ and go to step 5.
5. If $|x_2 - x_1| < \varepsilon$, stop with the root $R = \dfrac{x_1 + x_2}{2}$, other wise go to step 2.

4.4. Golden section method

The out line of the method is as follows :

1. Set the initial interval $[x_1 \, , \, x_2]$ and find $T = 0.382 \, L_0$.
2. Find the new interval $[x_3 \, , \, x_4]$:

$$x_3 = x_1 + T \qquad \text{and} \qquad x_4 = x_2 - T$$

3. Find the function value of x_3 and x_4, i.e. f_3 and f_4.
4. Check if $f_3 > f_4$ set $x_1 = x_3$ and go to step 5, else set $x_2 = x_4$ and go to step 5.

1. If $|x_2 - x_1| < \varepsilon$, stop with the root $R = \dfrac{x_1 + x_2}{2}$, other wise go to step 2.

Example :

The function :

$$f(x) = \sqrt{(x-2)^2 + 1} \ .$$

Using **Fibonacci** method, with the $[-3 , 10]$ and $n = 6$.

Solution :

Since:

$$f(x) = \sqrt{(x-2)^2 + 1} \quad , \quad [-3 , 10] \quad , \quad n = 6$$

Then :

$$L_0 \ = \ b - a \ = \ 10 - (-3) \ = \ 13$$

and :

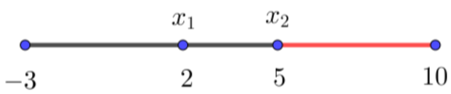
$$L_2^* \ = \left(\frac{F_{n-2}}{F_n} \right) L_0 \ = \frac{5}{13} \ 13 = \ 5$$

We find x_1 and x_2 from as :

$$x_1 \ = \ a + L_2^* \ = \ -3 + 5 = 2 \quad ; \quad f(x_1) = 1$$
$$x_2 \ = b - L_2^* \ = 10 - 5 \ = 5 \quad ; \quad f(x_2) = 3.162$$

Now,

$$1 \ = \ f(x_1) \ < \ f(x_2) = 3.162$$



$$x_3 \ = \ x_0 + (x_2 - x_1) = -3 + (5-2) = 0 \quad ; \quad f(x_3) = 2.236$$

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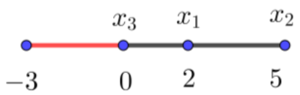
$$1 \ = \ f(x_1) \ < \ f(x_3) = 2.236$$



$$x_4 \ = \ x_2 - (x_1 - x_3) = 5 - (2-0) = 3 \quad ; \quad f(x_4) = 1.414$$

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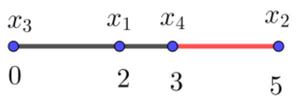
$$1 \;=\; f(x_1) \;<\; f(x_4) = 1.414$$



$$x_5 \;=\; x_3 + (x_4 - x_1) = 0 + (3-2) = 1 \quad ; \quad f(x_5) = 1.414$$

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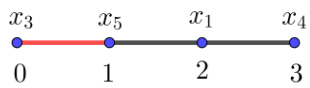
$$1 \;=\; f(x_1) \;<\; f(x_5) = 1.414$$



$$x_6 \;=\; x_4 - (x_1 - x_5) = 3 - (2-1) = 2 \quad ; \quad f(x_6) = 1.0$$

Example :

By using **Golden Section** method solve:

$$f(x) = \sqrt{(x-2)^2 + 1} \;.$$

with the $[-3\,,10\,]$ and $n = 6$.

Solution :

Since:

$$f(x) = \sqrt{(x-2)^2 + 1} \quad , \quad [-3\,,10\,] \quad , \quad n = 6$$

Then :

$$L_0 \;=\; b - a \;=\; 10 - (-3) \;=\; 13$$

and :

$$L_2^* = \frac{1}{\gamma^2} L_0 = (0.618)^2 \;\; L_0 = (0.382)\, L_0 = (0.382)(13) = 4.965$$

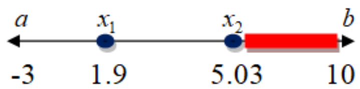
We find x_1 and x_2 from as :

$$x_1 = a + L_2^* = -3 + 4.965 = 1.965 \quad ; \quad f_1 = f(x_1) = \sqrt{(1.965-2)^2 + 1} = \sqrt{1.0012} = 1.0006$$

$$x_2 = b - L_2^* = 10 - 4.965 = 5.035 \quad ; \quad f_2 = f(x_2) = \sqrt{(5.035-2)^2 + 1} = \sqrt{10.211} = 3.1955$$

Now,

$$1.0006 \;=\; f(x_1) \;<\; f(x_2) = 3.1955$$



$$x_3 = a + (x_2 - x_1) = -3 + (5.035 - 1.965) = 0.07 \;; \quad f_3 = 2.1736$$

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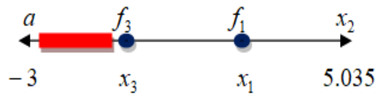
$\because \quad f_3 > f_1$



$$x_4 = b - (x_1 - x_3) = 5.035 - (1.965 - 0.07) = 3.14\ ; \quad f_4 = 1.5164$$

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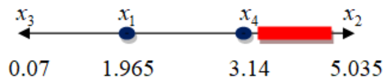
$\because \quad f_4 > f_1$



$$x_5 = a + (x_4 - x_1) = 0.07 + (3.14 - 1.965) = 1.245\ ; \quad f_5 = 1.253$$

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$\because \quad f_5 > f_1$

$$x_6 = b - (x_1 - x_5) = 3.14 - (1.965 - 1.245) = 3.14 - 0.72 = 3.068\ ; \quad f_6 = 1.463$$

$$\therefore \frac{L_6}{L_0} = \frac{x_6 - x_5}{L_0} = \frac{1.463 - 1.253}{13} = \frac{0.21}{13} = 0.0161$$