Exercise:

Consider the primal problem:

$$\left.\begin{array}{ll} Min & z = x_1 + x_2 \\ s.t. & 2x_1 + x_2 \ge 8 \quad ....(L_1) \\ & 3x_1 + 7x_2 \ge 21 \quad ....(L_2) \\ & x_1 \ge 0,\ x_2 \ge 0 \end{array}\right\}$$

Primal Lpp (النموذج الاولي)

a) Set up the dual.

b) Solve the primal and the dual by graphical method and verify that:

$$Min \quad Z = Max \quad V$$

Solution:

a) The dual is:

$$\left.\begin{array}{ll} Max & v = 8y_1 + 21y_2 \\ s.t. & 2y_1 + 3y_2 \le 1 \\ & y_1 + 7y_2 \le 1 \\ & y_1 \ ,\ y_2 \ge 0 \end{array}\right\}$$

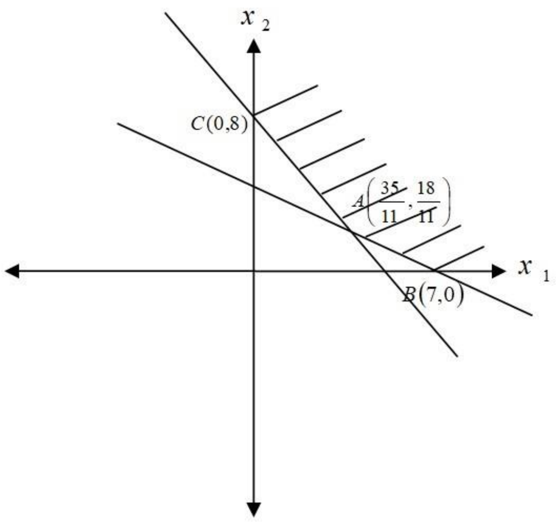
Dual Lpp (النموذج الثاني))

b) 1- solving the primal graphically:

$$\left.\begin{array}{ll} Min & z = x_1 + x_2 \\ s.t. & 2x_1 + x_2 \ge 8 \quad ....(L_1) \\ & 3x_1 + 7x_2 \ge 21 \quad ....(L_2) \\ & x_1 \ge 0,\ x_2 \ge 0 \end{array}\right\}$$

L_1 : (0 , 8) − (4 , 0)

L_2 : (0 , 3) − (7 , 0)



The solutions are :

$$A\left(\frac{35}{11},\frac{18}{11}\right) \ , \quad Z_A=\frac{35}{11}+\frac{18}{11}=\frac{53}{11}$$
$$B\ (7,0)\ , \qquad Z_B=7$$
$$C\ (0,8)\ , \qquad Z_C=8$$

<u>The optimal solution is :</u>

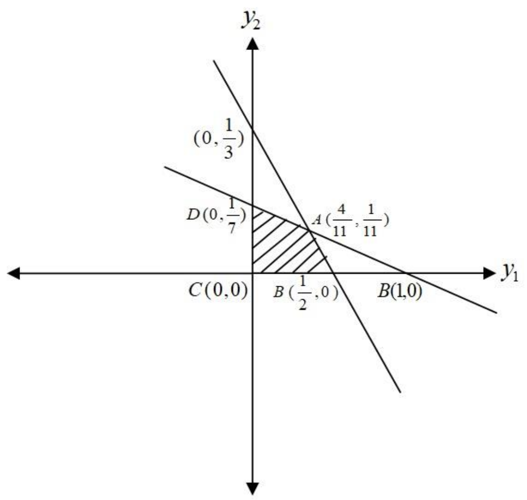
$$x_1=\frac{35}{11} \quad , \quad x_2=\frac{18}{11} \quad , \quad Z_A=\frac{35}{11}+\frac{18}{11}=\frac{53}{11}$$

<u>ملاحظة</u>: كلمة (Solve) تعني أن المسألة متكاملة ولها حل (يجب أن يكون لها حل)

2. For the dual,

$$\left.\begin{array}{ll} Max & v=8y_1+21y_2 \\ s.t. & 2y_1+3y_2\le 1 \\ & y_1+7y_2\le 1 \\ & y_1 \ , \ y_2\ge 0 \end{array}\right\}$$

$$L_1 \ : \ (0\ ,\ \frac{1}{3})-(\frac{1}{2}\ ,\ 0)$$
$$L_2 \ : \ (0\ ,\ \frac{1}{7})-(1\ ,\ 0)$$



The solutions are :

$$A\left(\frac{4}{11},\frac{1}{11}\right) \ , \quad Z_A=\frac{32}{11}+\frac{21}{11}=\frac{53}{11}$$
$$B\ (\frac{1}{2},0)\ , \qquad Z_B=7$$
$$C\ (0,0)\ , \qquad Z_C=0$$
$$D\ (0,\frac{1}{7})\ , \qquad Z_D=0$$

<u>The optimal solution is :</u>

$$y_1 = \frac{4}{11} \quad , \quad y_2 = \frac{1}{11} \quad , \quad Z_A = \frac{35}{11} + \frac{18}{11} = \frac{53}{11}$$

Theorem:

1. The value of the simplex multiplier associated with the optimal solution of the dual are the values of the variables for the optimal solution of the primal.

2. The primal values of the dual variables are negative of the simplex multipliers associated with the optimal solution of the primal.

$$1.\ \pi_j^* \cong x_j \quad ,\forall j \qquad 2.\ y_j \cong -\pi_j \quad ,\forall j$$

Exercise:

In the previas example find the simplex multipliers for the primal and the dual, then verify the above theorem with its two parts.

Solution:

<u>For the primal :</u>

$$\left.\begin{array}{ll} Min & z = x_1 + x_2 \\ s.t. & 2x_1 + x_2 \geq 8 \quad ....(\ \pi_1) \\ & 3x_1 + 7x_2 \geq 21 \quad ....(\ \pi_2) \\ & x_1 \geq 0,\ x_2 \geq 0 \end{array}\right\}$$

Then we get :

$$1 + 2\pi_1 + 3\pi_2 = 0$$
$$1 + \pi_1 + 7\pi_2 = 0$$

The solution of above equations is :

$$\pi_1 = -\frac{4}{11} \quad , \quad \pi_2 = -\frac{1}{11}$$

<u>For the dual put in</u>

$$
\left.\begin{array}{ll}
Min & v=-8y_1-21y_2 \\
s.t. & 2y_1+3y_2\le 1 \quad \ldots\ldots\ldots\ldots(*\pi_1^*) \\
 & y_1+7y_2\le 1 \quad \ldots\ldots\ldots\ldots(*\pi_2^*) \\
 & y_1 \ , \ y_2\ge 0
\end{array}\right\}
$$

Then we get :

$$
\begin{array}{l}
-8+2\pi_1^* + \quad \pi_2^* = 0 \\
-21+3\pi_1^* + 7\pi_2^* = 0
\end{array}
$$

The solution of above equations is :

$$\pi_1^* = \frac{35}{11} \quad , \quad \pi_2^* = \frac{18}{11}$$

*قيمة π_j^* في الـ dual تعطينا قيمة x_j في الـ primal مع الإشارة السالبة.

*كذلك π_j مع الإشارة السالبة تعطينا y_j.

*أذا كانت عدد المعادلات كبيرة مع المتغيرات نستخدم طريقة الـ dual لإيجاد الحل حيث أن من الصعب استخدام طريقة الـ graphically.

****

Exercise:

$$
\begin{array}{ll}
Min & z = 3x_1-2x_2+x_3 \\
s.t. & \left.\begin{array}{l} 2x_1-3x_2+x_3 = 1 \\ 2x_1+3x_2 \quad \ge 8 \end{array}\right\} \\
 & x_1 \ , \ x_2 \ , \ x_3 \ \ge 0
\end{array}
$$

(1) Dual.

(2) $\pi_j \ , \ \pi_j^*$.

(3) $Min \ \ z = Max \ \ v$