المقارنة بين اسلوبي المعاينة العشوائية الطبقية والمعاينة العشوائية البسيطة

في هذه الفقرة سنقوم ببرهنة اي من الاسلوبين ادق في التقدير من الاخر :

نظرية : في حالة اهمال كسر المعاينة $(f = 0)$ ،اثبت ان :

$$V[\bar{y}_{Ran.}] \geq V[\bar{y}_{Prop.}]$$

Proof :

$$\because \sigma^2_{(y)} = \sigma^2_{(w)} + \sigma^2_{(b)} = \frac{1}{N}\sum_{h=1}^{L} N_h \sigma_h^2 + \frac{1}{N}\sum_{h=1}^{L} N_h (\bar{Y}_h - \bar{Y})^2$$

بضرب المقدار بـ $\left(\frac{1}{n}\right)$ نحصل على :

$$\frac{1}{n}\sigma^2_{(y)} = \frac{1}{nN}\sum_{h=1}^{L} N_h \sigma_h^2 + \frac{1}{nN}\sum_{h=1}^{L} N_h (\bar{Y}_h - \bar{Y})^2$$

$$V[\bar{y}_{Ran.}] = V[\bar{y}_{Prop.}] + \frac{1}{nN}\sum_{h=1}^{L} N_h (\bar{Y}_h - \bar{Y})^2$$

$$V[\bar{y}_{Ran.}] = V[\bar{y}_{Prop.}] + \text{positive value}$$

توضيح :

$$\because V[\bar{y}_{Ran.}] = \frac{1-f}{n}\sigma^2_{(y)} \quad , \quad \because f = 0 \quad , \Rightarrow V[\bar{y}_{Ran.}] = \frac{1}{n}\sigma^2_{(y)}$$

$$\because V[\bar{y}_{Prop.}] = \frac{1}{N}\sum_{h=1}^{L} \frac{N_h \sigma_h^2}{n} - \frac{1}{N^2}\sum_{h=1}^{L} N_h \sigma_h^2$$

$$= \sum_{h=1}^{L} N_h \sigma_h^2 \left[\frac{1}{nN} - \frac{1}{N^2}\right] = \sum_{h=1}^{L} N_h \sigma_h^2 \left[\frac{N-n}{nN^2}\right] = \frac{\sum_{h=1}^{L} N_h \sigma_h^2}{nN}[1-f]$$

$$\because f = 0$$

$$\therefore V[\bar{y}_{Prop.}] = \frac{\sum_{h=1}^{L} N_h \sigma_h^2}{nN}$$

نظرية : في حالة عدم اهمال كسر المعاينة $(f \neq 0)$ وان $N-1 = N$)،$(N_h - 1 = N_h)$
) اثبت انه ايضاً :

$$V[\bar{y}_{Ran.}] = V[\bar{y}_{Prop.}] + \text{positive value}$$

Proof :

$$\because \sigma_{(y)}^2 = \frac{\sum_{h=1}^{L}\sum_{i=1}^{N_h}(Y_{hi} - \bar{Y})^2}{N-1}$$

بإضافة وطرح $(\bar{Y}_h)$ داخل القوس وعزل الحدود نحصل على :

$$(N-1)\sigma_{(y)}^2 = \sum_{h=1}^{L}\sum_{i=1}^{N_h}[(Y_{hi} - \bar{Y}_h) + (\bar{Y}_h - \bar{Y})]^2$$

$$= \sum_{h=1}^{L}\sum_{i=1}^{N_h}(Y_{hi} - \bar{Y}_h)^2 + \sum_{h=1}^{L}\sum_{i=1}^{N_h}(Y_{hi} - \bar{Y}_h)(\bar{Y}_h - \bar{Y}) + \sum_{h=1}^{L}\sum_{i=1}^{N_h}(\bar{Y}_h - \bar{Y})^2$$

$$= \sum_{h=1}^{L}\sum_{i=1}^{N_h}(Y_{hi} - \bar{Y}_h)^2 + \sum_{h=1}^{L} N_h (\bar{Y}_h - \bar{Y})^2$$

$$\because \sigma_h^2 = \frac{\sum_{i=1}^{N_h}(Y_{hi} - \bar{Y}_h)^2}{N_h - 1} \Rightarrow \sigma_h^2 (N_h - 1) = \sum_{i=1}^{N_h}(Y_{hi} - \bar{Y}_h)^2$$

$$\therefore (N-1)\,\sigma_{(y)}^2 = \sum_{h=1}^{L}(N_h - 1)\,\sigma_h^2 + \sum_{h=1}^{L} N_h (\bar{Y}_h - \bar{Y})^2$$

بضرب طرفي المعادلة بـ $\left(\frac{(1-f)}{n(N-1)}\right)$ نحصل على :

$$\sigma_{(y)}^2 \frac{(1-f)}{n} = \frac{(1-f)}{n(N-1)}\left[\sum_{h=1}^{L} N_h \sigma_h^2 - \sum_{h=1}^{L}\sigma_h^2 + \sum_{h=1}^{L} N_h (\bar{Y}_h - \bar{Y})^2\right]$$

بإضافة وطرح المقدار $[\frac{1}{N}\sum_{h=1}^{L} N_h\sigma_h^2]$ داخل القوس للطرف الايمن نحصل على :

$$=\frac{(1-f)}{n(N-1)}[\sum_{h=1}^{L} N_h\sigma_h^2-\frac{1}{N}\sum_{h=1}^{L} N_h\sigma_h^2-\sum_{h=1}^{L}\sigma_h^2+\frac{1}{N}\sum_{h=1}^{L} N_h\sigma_h^2$$

$$+\sum_{h=1}^{L} N_h(\overline{Y}_h-\overline{Y})^2]$$

$$V(\overline{y}_{Ran.})=\frac{(1-f)}{n(N-1)}[\sum_{h=1}^{L} N_h\sigma_h^2(1-\frac{1}{N})-\sum_{h=1}^{L}\sigma_h^2(-1+W_h)$$

$$+\sum_{h=1}^{L} N_h(\overline{Y}_h-\overline{Y})^2$$

$$V(\overline{y}_{Ran.})=\frac{(1-f)}{n(N-1)}\sum_{h=1}^{L}\frac{N_h}{N}\sigma_h^2(N-1)$$

$$V[\overline{y}_{Ran.}]=V[\overline{y}_{Prop.}]+\text{positive value }+\frac{(1-f)}{n(N-1)}[\sum_{h=1}^{L}\sigma_h^2(W_h-1)+\sum_{h=1}^{L} N_h(\overline{Y}_h-\overline{Y})^2]$$

نظرية : في حالة اهمال كسر المعاينة اثبت ان : $V[\overline{y}_{Ney.}]\le V[\overline{y}_{Prop.}]$

Proof :

لنأخذ الحالة :

$$\because f=0\ \ V[\overline{y}_{Prop.}]-V[\overline{y}_{Ney.}]=\frac{(1-f)}{n}\sum_{h=1}^{L} W_h\sigma_h^2-[\frac{(\sum_{h=1}^{L} W_h\sigma_h)^2}{n}-\frac{\sum_{h=1}^{L} W_h\sigma_h^2}{N}]$$

$$\therefore V[\overline{y}_{Prop.}]-V[\overline{y}_{Ney.}]=\frac{\sum_{h=1}^{L} W_h\sigma_h^2}{n}-\frac{(\sum_{h=1}^{L} W_h\sigma_h)^2}{n}+\frac{\sum_{h=1}^{L} W_h\sigma_h^2}{N}$$

$$=\frac{N\sum_{h=1}^{L} W_h\sigma_h^2+n\sum_{h=1}^{L} W_h\sigma_h^2}{nN}-\frac{(\sum_{h=1}^{L} W_h\sigma_h)^2}{n}$$

$$= \frac{\sum_{h=1}^{L} W_h \sigma_h^2 (N+n)}{nN} - \frac{(\sum_{h=1}^{L} W_h \sigma_h)^2}{n} = \frac{(1+f)\sum_{h=1}^{L} W_h \sigma_h^2}{n} - \frac{(\sum_{h=1}^{L} W_h \sigma_h)^2}{n}$$

$$= \frac{\sum_{h=1}^{L} N_h \sigma_h^2}{Nn} - \frac{(\sum_{h=1}^{L} N_h \sigma_h)^2}{N^2 n} = \frac{1}{nN}[\sum_{h=1}^{L} N_h \sigma_h^2 - \frac{(\sum_{h=1}^{L} N_h \sigma_h)^2}{N}]$$

بالإمكان كتابة المقدار الاخير بالصيغة الاتية :

$$= \frac{1}{nN}[\sum_{h=1}^{L} N_h \sigma_h^2 - \frac{2}{N}(\sum_{h=1}^{L} N_h \sigma_h)^2 + \frac{\sum_{h=1}^{L} N_h}{N^2}(\sum_{h=1}^{L} N_h \sigma_h)^2]$$

$$= \frac{1}{nN}\sum_{h=1}^{L} N_h[\sigma_h^2 - \frac{2}{N}\sigma_h(\sum_{h=1}^{L} N_h \sigma_h) + \frac{1}{N^2}(\sum_{h=1}^{L} N_h \sigma_h)^2]$$

$$= \frac{1}{nN}\sum_{h=1}^{L} N_h[\sigma_h - \frac{1}{N}\sum_{h=1}^{L} N_h \sigma_h]^2$$

$$\therefore V[\bar{y}_{Prop.}] - V[\bar{y}_{Ney.}] = \frac{1}{nN}\sum_{h=1}^{L} N_h[\sigma_h - \bar{\sigma}]^2$$

$$\text{where } \bar{\sigma} = \frac{\sum_{h=1}^{L} N_h \sigma_h}{N} \text{ Like } \bar{Y} = \frac{\sum_{h=1}^{L} N_h \bar{Y}_h}{N}$$

$$\therefore V[\bar{y}_{Prop.}] = V[\bar{y}_{Ney.}] + \frac{1}{nN}\sum_{h=1}^{L} N_h[\sigma_h - \bar{\sigma}]^2$$

$$\therefore V[\bar{y}_{Prop.}] = V[\bar{y}_{Ney.}] + \text{Positive value}$$

$$\therefore V[\bar{y}_{Ney.}] \le V[\bar{y}_{Prop.}]$$