Iteration methods

We using iteration methods because we either cannot the procedure analytically or because the analytical methods is intractable.

6. Iteration Methods of One Dimension

We sometimes need to determine the maximum or minimum value of a one-variable nonlinear function.

The solution techniques for one-dimensional optimization problems be the derivative of the function is driven to zero. The algorithm is terminated when the derivative of the function is very close to zero.

We discussing the different algorithms (approaches) for finding the minimum of a function of one variable. These algorithms are follows :

6.1. Bisection method

Idea give an interval $[a\,,b]$ use $f^{/}=0$ to find a local minimizer. The out line of the technique is as follows :

1. Set two points $a\,,b$ such that $f^{/}(a)$, $f^{/}(b)$ are of opposite sign.
2. Find $c=\dfrac{a+b}{2}$, and calculate $f^{/}(c)$.
3. If $f^{/}(a)\ .\ f^{/}(b)<0$, then $b=c$ else $a=c$.
4. The iteration is terminated if $|a-b|<\varepsilon$ or $f^{/}(c)<\varepsilon,$ then c is minimum, otherwise go to 2.

Example:

Find the value of x that minimizes $f(x)=x^2+54/x$ in the interval [2, 5].

Solution:

Evaluate $f'(x)$ at 2 and 5, such that :

$$f'(2) = -9.501 \quad \text{and} \quad f'(5) = 7.841$$

are of opposite sign. Then the new point we get:

$$x_3 = \frac{x_1 + x_2}{2} = \frac{2+5}{2} = 3.5 \quad \text{and} \quad f'(3.5) = 2.591$$

Since $f'(3.5) = 2.591 > 0$, the right half of the search space is eliminated.

$$x_4 = \frac{x_1 + x_3}{2} = \frac{2+3.5}{2} = 2.750 \quad \text{and} \quad f'(2.750) = -1.641$$

Since $f'(2.750) = -1.641 < 0$. Then the new point we get :

$$x_5 = \frac{x_3 + x_4}{2} = \frac{3.5+2.750}{2} = 3.125 \quad \text{and} \quad f'(3.125) = 0.720$$

The $f'(x_5) = f'(3.125) = 0.720$, since $|f'(x_5)| = |0.720|$ not less than ε, iteration continued.

6.2. Newton's method

Aim, minimize a function $f(x)$ of a single real variable x. Use Newton's method to solve $f'(x) = 0$.

The out line of the technique is as follows :

1. Set x_0 , ε; where ε is smallest number.
2. Find a point $x_1 = x_0 - \dfrac{f'(x_0)}{f''(x_0)}$.
3. If $|x_1 - x_0| < \varepsilon$, then x_1 is the minimum else set $x_0 = x_1$, go to 2.

Example:

The function :

$$f(x) = x^2 + 54/x \ .$$

Using Newton's method, with the starting guess $x_0 = 1$.

Solution:

The first and second derivatives of function are:

$$f'(x) = 2x - 54x^{-2} \quad \text{and} \quad f''(x) = 2 + 108x^{-3}$$

Evaluate $f'(x)$ and $f''(x)$ at the point $x_0 = 1$, such that :

$$f'(1) = -52 \quad \text{and} \quad f''(1) = 110$$

We compute the next guess,

$$x_1 = x_0 - \frac{f'(x_0)}{f''(x_0)} = 1 - \frac{-52}{110} = 1.4727$$

The derivative computed using (1.4727) at this point is found to be $f'(1.4727) = -21.9526$.

Since $|f'(1.4727)| = |-21.9526|$ not less than ε, we increment k to 2 and go to Step 2. This completes one iteration of the Newton Raphson method.

The Second Iteration needed to : $f''(1.4727) = 35.8128$

$$x_2 = x_1 - \frac{f'(x_1)}{f''(x_1)} = 1.4727 - \frac{-21.9526}{35.8128} = 1.4727 + 0.6130 = 2.0857$$

Since at every iteration first and second order derivatives are evaluated, a total of three function values are evaluated for every iteration.