* The rank of a matrix :- (r)

The rank of the matrix depends on the determinant of that matrix.

rank of the matrix

- $m \neq n$
  Non-square matrix
  - By deleting a row or column from the original matrix to find its determinant value
    - $|A| = 0$
      $r(A) < \min(m,n)$
    - $|A| \neq 0$
      $r(A) = \min(m,n)$
- $m = n$
  squar matrix
  - $|A| = 0$
    $r(A) < m$
    or $< n$
  - $|A| \neq 0$
    $r(A) = m$
    or $= n$

EX.: Find the rank for the following matrixes.

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 3 & 2 \\ -3 & -2 \end{bmatrix}$$

Sol:

$A_{2\times 2}$ $\therefore m = n = 2$ squar matrix

$|A| = 0 - 2 = -2 \neq 0$ $\therefore r(A) = m = n = 2$

$B_{2\times 2}$ $\therefore m = n = 2$ squar matrix

$|B| = -6 + 6 = 0$

So, will find a minor by deleting the row(i) and column(j) when ~~the~~ matrix is square.

delete (C_1 and R_1)

M_{11}

$B^{*} = [-2]_{1\times 1}$ $\therefore |B^{*}_{1\times 1}| = -2 \neq 0$

$\therefore r(B) = 1$.

Ex:- Find the rank of the following matrices

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -2 & -4 \\ 4 & 0 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 4 & 6 \\ 2 & 3 \\ -4 & -6 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Sol.:

$$|A| = \left|\begin{matrix} 1 & 2 & -1 \\ 3 & -2 & -4 \\ 4 & 0 & -5 \end{matrix}\right| \begin{matrix} 1 & 2 \\ 3 & -2 \\ 4 & 0 \end{matrix}$$

$$= [10 + (-32) + 0] - \{8 + 0 - 30\} = -22 + 22 = 0$$

So, minor will be found by deleting the row (1) and column (2) when the matrix is square

M_{12}

$$|A^*_{12}| = \left|\begin{matrix} 3 & -4 \\ 4 & -5 \end{matrix}\right| = -15 - (-16) = 1 \neq 0$$

$\therefore \; r(A) = 2$

$r(A) < m = 3 \quad \Rightarrow r(A) = 2 < m = 3$

$|B|_{2\times3}$ Since matrix(B) is not square (horizontal), only column (j) is deleted to convert it to a square matrix
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and then find the determinant.

delete C_1

$$B^*_{2\times2} = \begin{bmatrix} 3 & 2 \\ 0 & -1 \end{bmatrix} \; ; \; |B^*_{2\times2}| = \begin{vmatrix} 3 & 2 \\ 0 & -1 \end{vmatrix} = -3 \neq 0$$

$\therefore r(B) = 2$

$r(B) = \min(m, n)$

$= \min(2, 3)$

$r(B) = m = 2$

$|C|_{3\times2}$ Since matrix (C) is not square (vertical), only row (i) is deleted to convert it to a square matrix and then find the determinant.

delete R_3

$$C^*_{2\times2} = \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix} \rightarrow |C^*| = 12 - 12 = 0$$

delete R_2

$$|C^{*}_{2\,2\times2}| = \begin{vmatrix} 4 & 6 \\ -4 & -6 \end{vmatrix} = -24 + 24 = 0$$

delete(R_1)

$$|C^{*}_{3\,2\times2}| = \begin{vmatrix} 2 & 3 \\ -4 & -6 \end{vmatrix} = -12+12=0$$

since (C_1^*, C_2^* and C_3^*) all of them give the value of the determinant equal to zero, so minor will be found for $C_1^* = \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$

delete C_1, R_1

$|C_1^{**}| = |3| = 3 \neq 0$

$\therefore r(C) = 1$

$r(c) \leq \min(\overset{3}{m}, \overset{2}{n})$

$r(c) \leq n = 2$

$\therefore r(c) = 1 < n = 2$

$|D| = 0 \quad \therefore r(D) = 0$

- Since matrix D is a zero matrix, its rank is always zero

<u>Remark</u>: If the matrix is zero, then its rank is always zero

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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