4) Poisson Distribution: توزيع بواسون

The poisson distribution appears in many natural and physical phenomena.

1- The number of misprints per page in a Large Text.
2- The number of accidents per unit of time (hour, day, week, month) on a highway.
3- The number of Telephone calls per of time received at some Switchboard

The poisson dist. can be derived as a limiting case of a binomial dist. under the following conditions:

1- The number of trials n is very Large.
2- The probability of success of each trial p is small enough so that $np = \lambda$ is finite.

$$P(X) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & , x = 0, 1, 2, \dots \\ 0 & , o.w. \end{cases}$$

$X \sim Po(\lambda)$, λ: is parameter distribution

$$e^{\lambda} = \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = 1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \dots$$ دالة أسية

$$\sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \dots\right]$$

$$= e^{-\lambda} \cdot e^{\lambda} = e^{0} = 1 \Rightarrow P(x) \text{ is p.d.f.}$$

EX:
A student makes on the average, one typing error on each page, what is the prob. that the student would make exactly 2 typing errors in a 3 page term paper?
طالب يكتب معدل خطأ مطبعي واحد لكل صفحة ما هو احتمال ان الطالب يكتب بالضبط مرتين (خطأ مطبعي) في ثلاث صفحات.

Sol:

$n = 3$ عدد الصفات

$p = 1$ خطأ واحد في كل صفحة

$\lambda = np = 3 \times 1 = 3$

$\therefore X \sim P_o(3) \Rightarrow$

$$P(x) = \frac{e^{-3}\, 3^x}{x!} \quad , \quad X = 0, 1, 2 \quad \text{عدد الأخطاء}$$

$$\therefore P(X=2) = \frac{e^{-3}\, 3^2}{2!} = 0.224$$

احتمال أن الطالب يعمل خطأين مطبعيين في 3 صفحات.

EX: (2)

The average number of major fires per month in a certain city is 1.5, use the Poisson dist. to determine the prob. that there will be exactly one major fire in a period of two months.

Sol:

$\lambda = 1.5$, $X = 0, 1, \ldots$ عدد مرات الحريق

$$\therefore P(X) = \frac{e^{-1.5}\, 1.5^x}{x!}$$

$$\therefore P(X=1) = \frac{e^{-1.5} \cdot 1.5}{1!}$$