Linear Equation

- Linear Equation :

If a, and b are real numbers then $ax = b$ is called a linear equation in one variable (equation in first degree).

if a, b and c are real numbers then $ax + by = c$ is called a linear equation in two variables (unknowns).

if $a_1, a_2, \ldots, a_n$ and b are real numbers the $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$ is called a linear equation in (n) variables (unknowns).

A general system of m linear equations with (n) unknowns can be written as :-

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m$$

Here $x_1, x_2, \ldots x_n$ are the unknowns, $a_{11}, a_{12}, \ldots a_{mn}$ are the coefficients of the system, and $b_1, b_2, \ldots, b_m$ are the constant term.

* Methods for Solving equations in case of m=n

1- Solve a system of linear equations using matrices (factorization method) or (general rule)

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

or write as follows

$$AX = B$$

multiply both sides of the equation by A^{-1}

$$A^{-1} \cdot AX = A^{-1} \cdot B$$
$$I \cdot X = A^{-1} \cdot B$$
$$X = A^{-1} \cdot B$$

Ex. If you have

$$x_1 + x_2 = 3$$
$$2x_1 + x_2 = 6$$

Use the matrix method to solve the system of 2 equations.

Solution

$X = A^{-1} B$

$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

$A^{-1} = \frac{adj(A)}{|A|}$

$|A| = 1 - 2 = -1 \neq 0$

$adj(A) = \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{-1} \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}$

$X = A^{-1} \cdot B = -1 \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \end{bmatrix} = (-1) \begin{bmatrix} -3 \\ 0 \end{bmatrix}$

$\therefore X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$