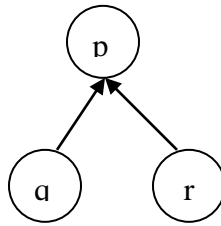
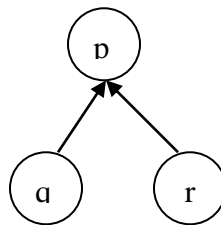


## AND/OR Graphs

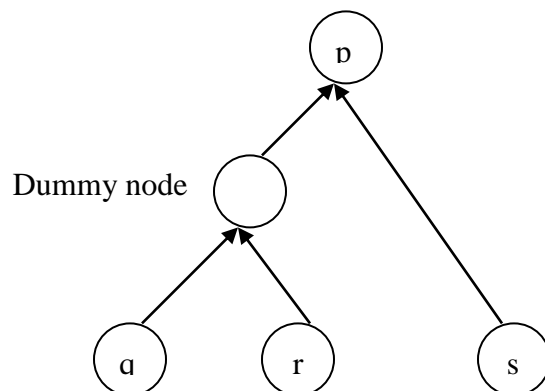
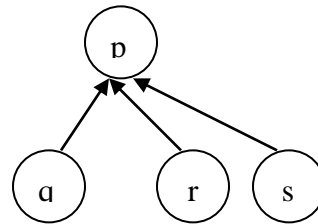
$$q \vee r \rightarrow p$$



$$q \wedge r \rightarrow p$$



$$\left. \begin{array}{l} q \wedge r \rightarrow p \\ s \rightarrow p \end{array} \right\} \equiv (q \wedge r) \vee s \rightarrow p$$



## - AND/OR Rule based deduction system:

Consider the problem

Goal:  $p \wedge (q \vee r)$

Clauses:

C1:  $\neg s \vee p$

C2:  $\neg u \vee s$

C3:  $u$

C4:  $\neg w \vee r$

C5:  $w$

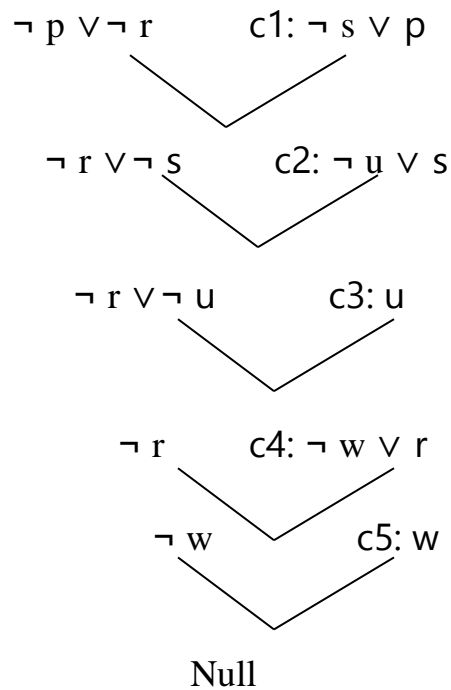
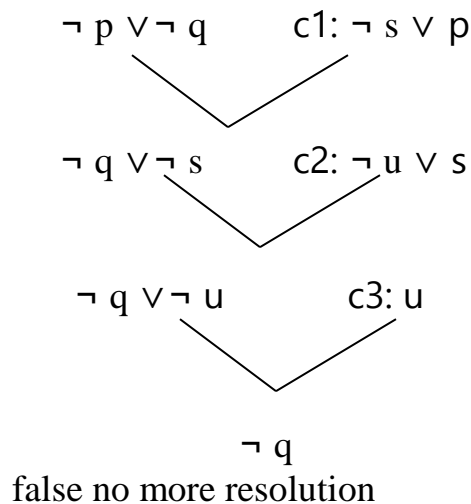
$\neg(p \wedge (q \vee r)) \rightarrow \neg p \vee \neg(q \vee r)$   
 $\rightarrow \neg p \vee (\neg q \wedge \neg r)$   
 $\rightarrow (\neg p \vee \neg q) \wedge (\neg p \vee \neg r)$

G1:  $(\neg p \vee \neg q)$

G2:  $(\neg p \vee \neg r)$

Take g1

Take g1



## Forward AND/OR rule based deduction system

1- facts: fact must be converted into a form called AND/OR form.

In this form, a fact must be represented as a conjunction of disjunction of literals. If the facts are not in this form. Then you have to convert to this form using the following steps:

- 1- eliminate  $\rightarrow$
- 2- reduce scope of negation.
- 3- standardize quantifiers.
  - 1- move all quantifiers to the left and skolemize  $\exists$  variables
  - 2- drop  $\forall$  quantifiers
  - 3- rename variables over the main conjunctions of the expression.

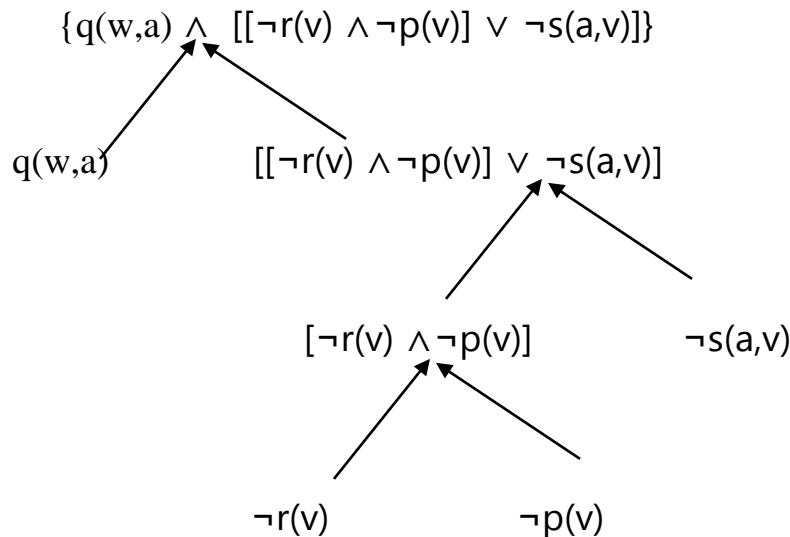
Ex:  $\exists u \forall v \{q(v,u) \wedge \neg [[r(v) \vee p(v)] \wedge s(u,v)]\}$

$\exists u \forall v \{q(v,u) \wedge [[\neg r(v) \wedge \neg p(v)] \vee \neg s(u,v)]\}$

$\forall v \{q(v,a) \wedge [[\neg r(v) \wedge \neg p(v)] \vee \neg s(a,v)]\}$

$\{q(w,a) \wedge [[\neg r(v) \wedge \neg p(v)] \vee \neg s(a,v)]\}$

Note: represent the fact expression in this AND/OR form as an AND/OR graph such that conjuncts are represented by OR nodes, disjuncts are represented by AND nodes.



2- rules: rules in the forward AND/OR deduction systems must have the form  $L \rightarrow W$ , where L is a literal, W is an expression which is an AND/OR form.

Quantification in rules must be over the entire rule to make the quantification on over the entire rule, perform the following steps:-

- 1- eliminate  $\rightarrow$
- 2- reduce scope of negation.
- 3- standardize quantifiers.
- 4- move all quantifiers to the left
- 5- skolemize  $\exists$  variables
- 6- drop  $\forall$  quantifiers
- 7- convert back to implication form using

$$\neg p \vee q \equiv q \rightarrow p$$

Ex:

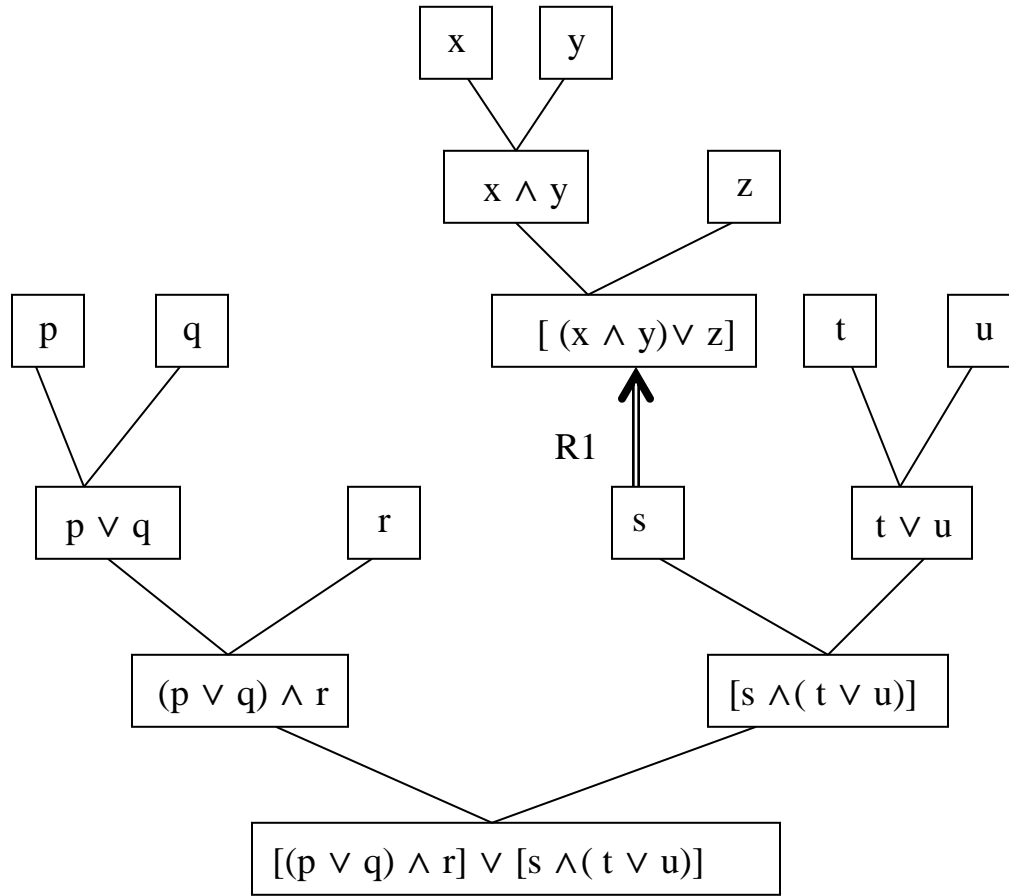
$$\begin{aligned} &(\forall x) \{[(\exists y) (\forall z) p(x,y,z)] \rightarrow \forall u q(x,u)\} \\ &(\forall x) \{\neg [(\exists y) (\forall z) p(x,y,z)] \vee \forall u q(x,u)\} \\ &(\forall x) \{(\forall y) (\exists z) \neg p(x,y,z) \vee \forall u q(x,u)\} \\ &(\forall x) (\forall y) (\exists z) (\forall u) \{\neg p(x,y,z) \vee q(x,u)\} \\ &(\forall x) (\forall y) (\forall u) \{\neg p(x,y,f(x,y)) \vee q(x,u)\} \\ &\quad \{\neg p(x,y,f(x,y)) \vee q(x,u)\} \\ &\neg p(x,y,f(x,y)) \rightarrow q(x,u) \} \end{aligned}$$

3- goal: the goal must be in the format of disjunction of literals.

Ex:

$$\text{Fact: } [(p \vee q) \wedge r] \vee [s \wedge (t \vee u)]$$

$$\text{Rule: } R1: s \rightarrow [(x \wedge y) \vee z]$$



After Applying R1, the following clauses are generated

$p \vee q \vee x \vee z$

$p \vee q \vee y \vee z$

$r \vee x \vee z$

$r \vee y \vee z$

$p \vee q \vee s$

$p \vee q \vee t \vee u$

$r \vee s$

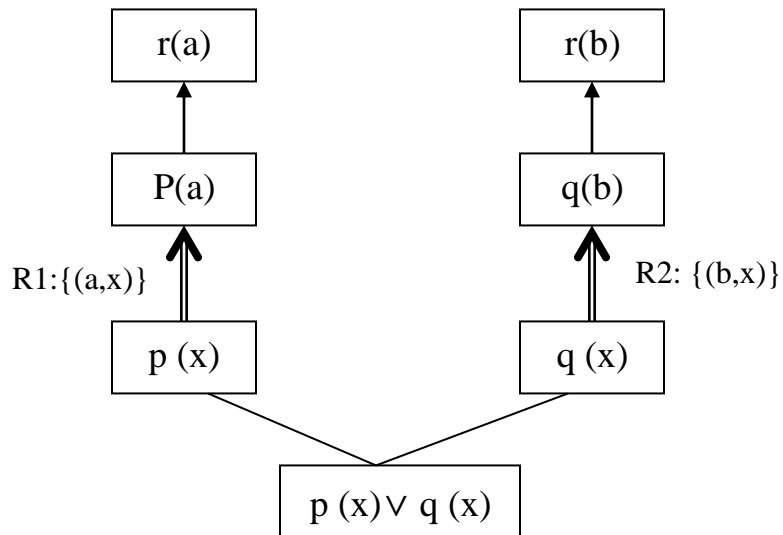
$r \vee t \vee u$

Ex: with variables

Fact:  $p(x) \vee q(x)$

Rules: R1:  $p(a) \rightarrow r(a)$

R2:  $q(b) \rightarrow r(b)$



Unifier composition =  $\{(a,x), (b,x)\}$

This is not a consistent solution