The Binomial Distribution

Modeling and Simulation

4th Stage

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Introduction

If we have a set S contains ***n*** distinct elements, and we have to arrange or select ***r*** elements extracted from the set S according to specified rules, so we used combinatorics.

* Note:

For arranging purpose, we have to use Combinations, and for selecting purpose, we have to use Permutaions.

Permutaion

A permutation is an arrangement of objects in a definite order. The members or elements of sets are arranged here in a sequence or linear order. For example, the permutation of set A={1,6} is 2, such as {1,6}, {6,1}. As you can see, there are no other ways to arrange the elements of set A. When we look at the schedules of trains, buses and the flights we really wonder how they are scheduled according to the public's convenience. Of course, the permutation is very much helpful to prepare the schedules on departure and arrival of these.

Permutaions

Permutaions: An ordered arrangement of ***r*** elements of a set containing ***n*** elements is called an r-permutaion of n elements and is denoted by P(n,r) or ${}^{n}P_{r}$, where:

r is less than or equal to ***n***.

$p(n,r)= n(n-1)(n-2)(n-3)......(n-r+1)$

$$\boxed{\begin{array}{l} \mathbf{P(n,r) = n!/(n-r)!} \\ \\ \mathbf{P(n,n) = n!} \end{array}}$$

Tasks on Permutaions

1- Number of permutaions of n objects (when repitition is not allowed):

The numeber of diffirenet arrangements (permutaions) of n distinct objects taken all at a time is represented by:

$$\mathbf{P(n,n) = n!}$$

2-Number of permutaions of r objects among n distinct(different) objects(when repitition is not allowed):

Suppose we are given n distinct objects and wish to arrange r of the r of the objects from the given set, we have to use the rule:

Tasks on Permutaions

3- Permutation when repetition is allowed:

We can easily calculate the permutation with repetition. The permutation with repetition of objects can be written using the exponent form.

$$\mathbf{n \times n \times n \times \ldots\ldots (r\ times) = n^r}$$

Tasks on Permutaions

Ex1: How many 3 letter words with or without meaning can be formed out of the letters of the word SAVED when repetition of words is allowed?

Sol: The number of objects, in this case, is 5, as the word SAVED has 5 alphabets.

and $r = 3$, as 3-letter word has to be chosen.

Thus, the permutation will be:

Permutation (when repetition is allowed) $= 5^3 = 125$

Tasks on Permutaions

Ex2: For a password consists of 3 digits, How many passwords can be extracted when repetition is allowed?

Sol:

The total number of digits = 10, (0-9)

So, $10^3 = 1000$ passwords

Ex3: how many 3 letter words can be created, if Repetition is allowed?

Sol: $\mathbf{26^3 = 17576}$

Permutaions

4- Number of permutaions of n objects (multi-sets permutaion):

Permutation of n different objects when p_1 objects among 'n' objects are similar, p_2 objects of the second kind are similar, p_3 objects of the third kind are similar ……… and so on, p_k objects of the kth kind are similar and the remaining of all are of a different kind, $\mathbf{n!/n_1!*n_2!*n_3!*......*n_k!}$

5- Circular permutaion:

The total nimber of ways of arranging n objects in a circle represented by the following rule:

$$\textbf{circle} = \mathbf{(n-1)!}$$

Examples

Ex4:

How many different strings of length 4 can be formed using the letters of the word “great” when repitition is not allowed ?

Sol:

The give word has 5 letters,

so, $n = 5$

The number of strings of length 4 can br formed using the letters of the word

$P(n,r) = P(5,4) = 5!/(5-4)!$

$= 5!/1!$

$= 5*4*3*2*1$

$= 120$

Examples

Ex5:

How many words of 3 distinct letters can be formed from the word PASCAL? (repitition is not allowed)

The word PASCAL consists of 6 leters

So, $n=6$

The number of ways 3 distinct letters can be formed from the word PASCAL

$P(n,r) = P(6,3)$

$= 6!/(6-3)!$

$6!/3! = 6*5*4*\cancel{3}!/\cancel{3}! = 6*5*4 = 120$

Examples

Ex6:

Find the number of permutaions of the letters of the word ENGINEERING.

Sol: The given word ENGINEERING has 11 letters out of which E are 3, N are 3, I are 2, G are 2 and R is 1

$n!/n_1!*n_2!*n_3!*n_4!*n_5!$

$11!/3!*3!*2!*2!*1!$

$= 11*10*9*8*7*\cancel{6}*5*\cancel{4}*\cancel{3!}/\cancel{3!}*\cancel{3}*\cancel{2}*\cancel{2}*\cancel{2}*1$

$= 11*10*9*8*7*5$

$= 277200$

Applications

Some applications of Permutations :

For Permutaions:

1- Phone numbers

In phone no. there are for example 8 digits in which first 2 represent the area code and next 2 digit represent the city code so the first 4 digits are fixed.. next 4 digits are permuted...to form the new no.

2- Car plate numbers

First 2 digits represent the state code. eg-DL for Delhi, GA for Goa

etc. then next a digit is assigned for type of vehicle. eg- C for Car, T for Taxi. then next 4 digits represent numbers which are permuted and are unique for every car.

Combinations

A combination is a selection of items from a collection, such that the order of selection does not matter (unlike permutations).

An ordered selection of r elements of a set containing n distinct elements is called r-combination of n elements and is denoted by C(n,r) or $^{n}C_{r}$

C(n,r) = n!/r!(n-r)!

C(n,n) = 1

Combinations

Note:

So, for choosing objects from a set, we use **Combination**, and for arranging objects in a determined set, we use **Permutaion**.

Combinations

Combination Formula

A combination is a grouping or subset of items.
For a combination, the order does not matters.

$$C(n,r) = {}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

Number of items in set

Number of items selected from the set

Combination examples

Ex1:

Picking a team of 3 persons from a group of 10.

sol:

$C(n,r) = n!/r!(n-r)!$

$C(10,3) = 10!/\ 3!*7!$

$= 10*9*8*\cancel{7!}/3!*\cancel{7!}$

$= 10*\cancel{9}*\cancel{8}/\cancel{3}*\cancel{2}$

$= 10*3*4 = 120$

Examples

Ex2:

In how many ways can a coach choose three players from among five players?

There are 5 players to be taken 3 at a time.

Using the formula:

C(n,r) = n!/r!(n-r)!

C(5,3) = 5!/3!*2!

$= 5*4*\cancel{3}!/\cancel{3}!*2!$

= 20/2 = 10

The coach can choose the plyers in 10 ways.