Importance of Mathematical Modeling

A mathematical model, as stated, is a mathematical description of a real life situation. So, if a mathematical model can reflect or mimic the behavior of a real-life situation, then we can get a better understanding of the system through proper analysis of the model using appropriate mathematical tools.

Moreover, in the process of building the model, we discover various factors that govern the system, that are most important to the system, and that reveal how different aspects of the system are related. The importance of mathematical modeling in physics, chemistry, biology, economics, and even industry cannot be ignored. Mathematical modeling in basic sciences is gaining popularity, mainly in biological sciences, economics, and industrial problems. For example, if we consider mathematical modeling in the steel industry, many aspects of steel manufacture, from mining to distribution, are susceptible to mathematical modeling. In fact, steel companies have participated in several mathematics-industry workshops, where they discussed various problems and obtained solution through mathematical modeling – problems involving control of ingot cooling, heat and mass transfer in blast furnaces, hot rolling mechanics, friction welding, spray cooling and shrinkage in ingot solidification, to mention a few [115]. Similarly, mathematical modeling can be used (i) to study the growth of plant crops in a stressed environment, (ii) to study mRNA transport and its role in learning and memory, (iii) to model and predict climate change, (iv) to study the interface dynamics for two liquid films in the context of organic solar cells, (v) to develop multi-scale modeling in liquid crystal science and many more. For gaining physical insight, analytical techniques are used. However, to deal with more complex problems, numerical approaches are quite handy. It is always advisable and useful to formulate a complex system with a simple model whose equation yields an analytical solution. Then, the model can be modified to a more realistic one that can be solved numerically. Together with the analytical results for simpler models and the numerical solution from more realistic models, one can gain maximum insight into the problem.

Latest development of Mathematical Modeling

Mathematical modeling is an area of great development and research. In recent years, mathematical models have been used to validate hypotheses made from experimental data, and at the same time, the designing and testing of these models have led to testable experimental predictions. There are impressive cases in which mathematical models have provided fresh insight into biological systems, physical systems, decision-making problems, space models, industrial problems, economical problems, and so forth. The development of mathematical modeling is closely related to significant achievements in the field of computational mathematics. Consider a new product being launched by a company. In the development process, there are critical decisions involved in its launch such as timing, determining price, launch sequence. Experts use and develop mathematical models to facilitate such decision making. Similarly, in order to survive market competition, cost reduction is one of the main strategies for a manufacturing plant, where a large amount of production operation costs are involved. Proper layout of equipment can result in a huge reduction in such costs. This leads to a dynamic facility layout problem for finding equipment sites in manufacturing environments, which is one of the developing areas in the field of mathematical modeling [152]. Mathematical modeling also intensifies the study of potentially deadly flu viruses or Coronavirus (COVID-19) from mother nature and bio-terrorists. Mathematical models are also being developed in optical sciences [8], namely, diffractive optics, photonic band gap structures and waveguides, nutrient modeling, studying the dynamics of blast furnaces, studying erosion, and prediction of surface subsidence. In geosciences, mathematical models have been developed for talus. Talus is defined by Rapp and Fairbridge [128] as an accumulation of rock debris formed close to mountain walls, mainly through many small rockfalls. Hiroyuki and Yukinori [114] have constructed a new mathematical model for talus development and retreat of cliffs behind the talus, which was later applied to the result of a field experiment for talus development at a cliff composed of chalk. They developed the model which was in agreement with the field observations. There was tremendous development in the interdisciplinary field of applied mathematics in human physiology in the last decade, and this development still continues. One of the main reasons for this development is the researcher's improved ability to gather data, whose visualization has much better resolution in time and space than just a few years ago. At the same time, this development also constitutes a giant collection of data as obtained from advanced measurement techniques. Through statistical analysis, it is possible to find correlations, but such analysis fails to provide insight into the mechanisms responsible for these correlations. However, when it is combined with mathematical modeling, new insights into the physiological mechanisms are revealed. Mathematical models are being developed in the field of cloud computing to facilitate the infrastructure of computing resources in which large pools of systems (or clouds) are linked together via the internet to provide IT services.

Limitations of Mathematical Modeling

Sometimes although the mathematical model used is well adapted to the situation at hand, it may give unexpected results or simply fail. This may be an indication that we have reached the limit of the present mathematical model and must look for a new refinement of the real-world or a new theoretical breakthrough [129]. A similar type of problem was addressed in [8], which deals with Moire theory, involving the mathematical modeling of the phenomena that occur in the superposition of two or more structures (line gratings, dot screens, etc.), either periodic or not. In mathematical modeling, more assumptions must be made, as information about real-world systems become less precise or harder to measure. Modeling becomes a less precise endeavor as it moves away from physical systems towards social systems. For example, modeling an electrical circuit is much more straightforward than modeling human decision making or the environment. Since physical systems usually do not change, reasonable past information about a physical system is quite valuable in modeling future performance. However, both social systems and environments often change in ways that are not of the past, and even correct information may be of less value in forming assumptions. Thus, to understand a model's limitations, it is important to understand the basic assumptions that were used to create it. Real-world systems are complex and a number of interrelated components are involved. Since models are abstractions of reality, a good model must try to incorporate all critical elements and interrelated components of the real-world system. This is not always possible. Thus, an important inherent limitation of a model is created by what is left out. Problems arise when key aspects of the real-world system are inadequately treated in a model or are ignored to avoid complications, which may lead to incomplete models. Other limitations of a mathematical model are that they may assume that the future will be like the past, input data may be uncertain, or the usefulness of a model may be limited by its original purpose. However, despite all these limitations and pitfalls, a good model can be formulated, if a modeler asks himself/herself the following questions about the model: (i) Does the structure of the model resemble the system being modeled? (ii) Why is the selected model appropriate to use in a given application? (iii) How well does the model perform? (iv) Has the model been analyzed by someone other than the model authors? (v) Is adequate documentation of the model available for all who wish to study it? (vi) What assumptions and data were used in producing model output for the specific application? (vii) What is the accuracy of the model output? One should not extrapolate the model beyond the region of fit. A model should not be applied unless one understands the simplifying assumptions on which it is based and can test their applicability. It is also important to understand that the model is not the reality and one should not distort reality to fit the model. A discredited model should not be retained, and one should not limit himself to a single model, as more than one model may be useful for understanding different aspects of the same phenomenon. It is imperative to be aware of the limitations inherent in models. There is no best model, only better models.

How to build Mathematical Model:

Step1 (the start):

Before we start formulating a model, we should be clear about our objectives. The objectives decide the further course of the task in two different ways: (i) Firstly, the amount of detail incorporated into the model depends on the purpose for which the model will be utilized. For example, in modeling the growth of skin cancer (melanoma) to act as an aid for oncologists, an empirical model containing terms for the most significant determinants of skin cancer growth may be quite adequate and the model can be viewed as a summary of the current understanding of melanoma (skin cancer). However, such a model is clearly of limited use as a research tool to investigate the growth of exocrine tumors (pancreatic cancer). (ii) Secondly, there should be a clear-cut division between the system to be modelled and its environment. Care should be taken that the division is well-made, which means that the environment may affect the dynamics of the system but the system does not affect the environment.

Step2(the assumption)

Once the system which needs to be modelled is decided, a basic framework of the model needs to be constructed. This will depend on the underlying assumption that we make in the construction of the model. It is to be noted that whenever assumptions are made, they will be considered as true while doing analysis of the system and results of such an analysis are only as valid as the assumptions. Thus, one of the fundamental differences between classical mechanics and relativity theory is that Newton assumed that mass is the universal constant, whereas Einstein considered mass as a variable. Let us consider the growth of a population. It is a common assumption that a population will grow at a rate that is proportional to its size, in absence of limiting factors. In continuous time, such a population growth will be represented by the differential equation

$dx / dt = rx$,

Step3 (Schematic or Flow Diagram)

Whether the system is simple or complex, it is always advisable not to simply jump from an assumption to an equation. A much more methodical step in the form of schematic or flow diagrams is desirable, which provides a visual aid to the problem. A flow diagram or schematic diagram is a pictorial representation depicting the flow of steps in a model. The elements of the schematic diagram are represented by circles, rectangles, diamonds, or other shapes, with lines and arrows representing connections between events and the direction or order in which they occurs. The schematic diagram shows a starting point, an ending or stopping point, along with the sequence and decision points. Since, it is easier to understand a process in a visual form than in a verbal description, a schematic or flow diagram is always useful to start a modeling process. Consider a susceptible-infected-recovered-susceptible (SIRS) model, where a person, susceptible to a particular disease, gets infected at a rate α when he/she comes in contact with an infected one. From the infected class, a person gets cured at a rate β and move to the recovered class. The recovered person is also susceptible to the disease at a rate δ. The schematic diagram for this scenario is given in fig. 1.1.

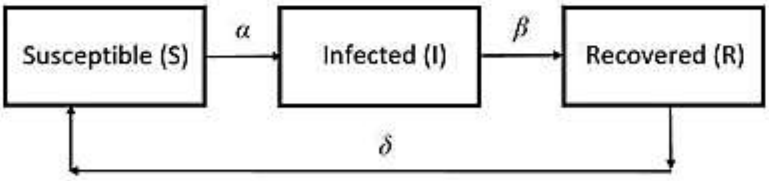

FIGURE 1.1: *The schematic diagram for susceptible-infected-recovered-susceptible (SIRS) model.*

Step4(Choosing Mathematical equations)

One can now choose the appropriate mathematical equations, once the structure of the model has been decided. A good starting point would be to look equations from literatures, where somebody else may have published an equation relating to the quantities you are interested in. But, it is necessary to proceed with caution as equations in the literatures may not be expressed exactly in the form required for the model.

Step5(Solving equations)

In general, obtaining an analytical solution of the mathematical model of a real-life problem is rare. If the model consists of linear system of differential equations, or if the model consists of just one differential equations (even non-linear), there is a good chance of obtaining analytical solutions. However, when models contain a system of non-linear differential equations, it is difficult to solve them analytically. Then, we use numerical methods to obtain approximate solutions, which can never have the same generality as analytical solutions, but numerical solutions of the model generally mimic the process described in the model.