6- The Hessian matrix of a function f which is defined by $G(x)$, or$\nabla^2 Q(x)$, is the matrix of second partial derivatives, i.e.

$$\nabla^2 Q(x) = \begin{bmatrix} \frac{\partial^2 Q(x)}{\partial x_1^2} \frac{\partial^2 Q(x)}{\partial x_1 \partial x_2} \cdots \frac{\partial^2 Q(x)}{\partial x_1 \partial x_n} \\ \frac{\partial^2 Q(x)}{\partial x_2 \partial x_1} \frac{\partial^2 Q(x)}{\partial x_2^2} \cdots \frac{\partial^2 Q(x)}{\partial x_2 \partial x_n} \\ \cdots\cdots \\ \cdots\cdots \\ \cdots\cdots \\ \frac{\partial^2 Q(x)}{\partial x_n \partial x_1} \frac{\partial^2 Q(x)}{\partial x_n \partial x_2} \cdots \frac{\partial^2 Q(x)}{\partial x_n^2} \end{bmatrix},$$

where the Hessian matrix is symmetric and positive define

7- We say that an optimization algorithm has ELS if $g_{k+1}^T d_k = 0$for $k = 1,2, \ldots n$

8- We say that an optimization algorithm satisfies the descent condition, if

$$d_k^T g_k < 0, \forall k \geq 1,$$

and hence $f_{k+1} < f_k$, for all k must be satisfied.

9- The search direction d_k is say the descent condition holds if it satisfy the condition

$$g_k^T d_k < 0 \qquad \forall\, k \geqslant 0$$

We say that an optimization algorithm satisfies the sufficient descent condition, if there exists a constant c>0, such that

$$g_k^T d_k \leqslant -c\|g_k\|^2 \text{ for all } k \geqslant 0.$$

General Nonlinear Constrained:

The general constrained minimization problem

min f(x)

s.t

$g_i \leq 0$ for i=1,2,……,p

$h_j = 0$ for j=1,2,……,m

$f : R^n \to R$ smooth function e_i is inequality constrained and c_j equality constrained. Usually as-summed to possess continuous second partial derivatives. The constraints are referred to as functional constraints.

Necessary and Sufficient Conditions for Constrain Optimization

The First Order Optimality Condition

1- Karush-Kuhn-Tucker (KKT) Necessary Conditions:

Let $\bar{x}$ be a feasible solution of (P) and let $I = \{i : g_i(\bar{x}) = 0\}$. Further, suppose that $\nabla g_i(\bar{x})$ for $i = 1,2,\ldots\ldots\ldots l$ and $\nabla g_i(\bar{x})$ for $i \in I$ are linearly independent. If $\bar{x}$ is a local minimum, there exists (u, v) such that

$$\nabla f(\bar{x}) + \nabla g(\bar{x})^T u + \nabla h(\bar{x})^T v = 0,$$

$$u \geq 0,$$

$$u_i g_i(\bar{x}) = 0, i = 1,\ldots\ldots\ldots\ldots m$$