2- Karush-Kuhn-Tucker (KKT) Sufficient Conditions:

Suppose that (P) is a convex problem. Let $\bar{x}$ be a feasible solution of (P), and suppose $\bar{x}$ together with multipliers (u, v) satisfies

$$\nabla f(\bar{x}) + \nabla g(\bar{x})^T u + \nabla h(\bar{x})^T v = 0,$$
$$u \geq 0,$$
$$u_i g_i(\bar{x}) = 0, i = 1, \ldots\ldots\ldots m$$

Then $\bar{x}$ is a global optimal solution of (P).

The Second Order Optimality Condition:

1- Karush-Kuhn-Tucker (KKT) Necessary Conditions

Suppose $\bar{x}$ is a local minimum of (P), and $\nabla c_i(\bar{x})$ for $i = 1,2, \ldots\ldots$, $\nabla e_i(\bar{x})$ for $i \in I$ are linearly independent. Then $\bar{x}$ must satisfy the KKT conditions. Furthermore, every d that satisfies:

$$\nabla e_i(\bar{x})^T d \leq 0, i \in I$$
$$\nabla c_i(\bar{x})^T d = 0, i = 1, \ldots\ldots\ldots l$$

must also satisfy

$$d^T \nabla^2_{xx} L(\bar{x}, u, v) d \geq 0$$

2- Karush-Kuhn-Tucker (KKT) Sufficient Conditions

Suppose the point $\bar{x} \in S$ together with multipliers (u, v) satisfies the KKT conditions. Let $I^+ = \{i \in I: u_i > 0\}$ and $I^0 = \{i \in I: u_i = 0\}$. Additionally, suppose that every $d \neq 0$ that satisfies

$$\nabla e_i(\bar{x})^T d = 0, i \in I^+,$$
$$\nabla e_i(\bar{x})^T d \leq 0, i \in I^0,$$

$$\nabla c_i(\bar{x})^T d = 0, i \in 1, \ldots\ldots. l$$

also satisfies

$$d^T \nabla_{xx}^2 L(\bar{x}, u, v) d > 0.$$

Then $\bar{x}$ is a strict local minimum of (P).