From (i) and (ii),
or

$$4 - 2x_1 = 8 - 2x_2$$

and from (iii), $x_2 + x_1 = 2$,
which give $x_1 = 0, x_2 = 2$ and $\lambda = 4$.
The sufficient conditions for determining whether the above solution results in maximization

EXAMPLE
Solve the following NLPP :

$$\begin{aligned} \text{Maximize} \quad Z &= 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2{x_2}^2, \\ \text{subject to} \quad x_1 + 2x_2 &= 2, \\ x_1, x_2 &\geq 0. \end{aligned}$$

Solution
The objective function as well as the constraint are differentiable w.r.t. x_1 and x_2.
The constraint can be replaced by another differentiable function such as,

$$x_1 + 2x_2 - 2 = 0.$$

The Lagrangean function can be written as

$$\mathrm{L}(x_1, x_2, \lambda) = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2 - \lambda(x_1 + 2x_2 - 2).$$

The necessary conditions for a maxima or minima of the objective function are

$$\frac{\partial L}{\partial x_1} = 0,$$
$$\frac{\partial L}{\partial x_2} = 0,$$
$$\frac{\partial L}{\partial \lambda} = 0,$$

or

$$4 - 4x_1 - 2x_2 - \lambda = 0,$$
$$6 - 2x_1 - 4x_2 - 2\lambda = 0,$$

and $-(x_1 + 2x_2 - 2) = 0.$

and

Solving (i) and (ii), we get $x_1 = 1/3$ which when substituted in (iii) gives $x_2 = 5/6$ and for this solution $\lambda = 1$.

EXAMPLE

Solve the following N.L.P.P.:

$$\begin{aligned} \text{Maximize} \quad & Z = 5x_1 + x_2 - (x_1 - x_2)^2, \\ \text{subject to} \quad & x_1 + x_2 = 4, \\ & x_1, x_2 \geq 0. \end{aligned}$$

Solution

The Lagrangean function for the above problem can be formed as

$$L(X, \lambda) = 5x_1 + x_2 - (x_1 - x_2)^2 - \lambda(x_1 + x_2 - 4)$$

THe necessary conditions for the objective function to be maximum or minimum are

$$\frac{\partial L}{\partial x_1} = 5 - 2(x_1 - x_2) - \lambda = 0,$$
$$\frac{\partial L}{\partial x_2} = 1 + 2(x_1 - x_2) - \lambda = 0,$$
$$\frac{\partial L}{\partial \lambda} = -(x_1 + x_2 - 4) = 0.$$