The solution of these equations yields

$$x_1 = \frac{5}{2}, x_2 = \frac{3}{2} \text{ and } \lambda = 3.$$

EXAMPLE

Solve the following N.L.P.P. by using Lagrangean multipliers method:

$$\begin{aligned} &\text{Minimize} && Z &&= x_1^2 + x_2{}^2 + x_3{}^2, \\ \text{subject to} \quad & 4x_1 + x_2{}^2 + 2x_3 && && = 14, \\ & x_1, x_2 && && \geq 0. \end{aligned}$$

Solution

The Lagrangean function for the given problem can be written as

$$\mathrm{L}(x_1, x_2, x_3, \lambda) = x_1^2 + x_2^2 + x_3^2 - \lambda(4x_1 + x_2^2 + 2x_3 - 14).$$

The necessary conditions for the above convex function to be minimum are

$$\frac{\partial \mathrm{L}}{\partial x_1} = 2x_1 - 4\lambda = 0,$$
$$\frac{\partial \mathrm{L}}{\partial x_2} = 2x_2 - 2\lambda x_2 = 0,$$
$$\frac{\partial \mathrm{L}}{\partial x_3} = 2x_3 - 2\lambda = 0,$$
$$\frac{\partial \mathrm{L}}{\partial \lambda} = -(4x_1 + x_2^2 + 2x_3 - 14) = 0.$$

From (i)

$$\lambda = \frac{x_1}{2} \text{ and from (iii) } \lambda = x_3.$$

From (ii) $x_2(1 - \lambda) = 0$ i.e., either $x_2 = 0$ or $\lambda = 1$.
When
$x_2 = 0$,
from (iv) $4(2\lambda) + 0 + 2\lambda = 14$ or $\lambda = 1.4$.
This gives the solution $x_1 = 2.8, x_2 = 0, x_3 = 1.4$,
with

When
from (i)
from (iii)
and from (iv) $8 + x_2^2 + 2 = 14$ or $x_2^2 = 4$
or
Since

$$Z_{\text{min}} = 9.8.$$

$$\lambda = 1,$$
$$x_1 = 2,$$

$x_3 = 1,$
$x_2 = \pm 2.$
$x_2 \geq 0, x_2 = -2$ is discarded.
∴ The second solution is $x_1 = 2, x_2 = 2, x_3 = 1$, giving $Z_{\min} = 9$.
Since $(x_1, x_2, x_3) = (2,2,1)$ gives the smaller value of $Z_{\min}$, it is the optional solution.