Ex Find the limit

$$\lim_{x\to\infty} \frac{2x}{2x} = \lim_{x\to\infty} 1 = 1$$

∴ The limit at $x\to\infty$ is exist

Find the limit

$$\lim_{x\to\infty} \frac{x^3+x^2+x}{2x^3+1} = \frac{\infty}{\infty}$$

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$$= \lim_{x\to\infty} \frac{\frac{x^3}{x^3}+\frac{x^2}{x^3}+\frac{x}{x^3}}{\frac{2x^3}{x^3}+\frac{1}{x^3}}$$

$$= \lim_{x\to\infty} \frac{1+\frac{1}{x}+\frac{1}{x^2}}{2+\frac{1}{x^3}} = \frac{1+\frac{1}{\infty}+\frac{1}{\infty^2}}{2+\frac{1}{\infty^3}} = \frac{1}{2}$$

Ex $\lim_{x\to\infty} \frac{x^2+2x+5}{x^3+x}$

$$= \frac{\infty^2+2(\infty)+5}{(\infty)^3+(\infty)} = \frac{\infty}{\infty}$$

Using lopital we get :-

$$\lim_{x\to\infty} \frac{2x+2}{3x^2+1} = \frac{2(\infty)+2}{3(\infty)^2+1} = \underline{\underline{2}}$$

The Continuity

we Say that f(x) is Continuous if and only if Satisfy the following conditions :-

1. $f(a) \in R$ is defined

2. $\lim_{x \to a} f(x)$ exist

3. 1. = 2. or $f(a) = \lim_{x \to a} f(x)$

Ex 1. The f(x) is Continuous or not ?

$$f(x) = \begin{cases} 2x - 5 & ; \ x \neq 1 \\ 2 & ; \ x = 1 \end{cases}$$

Solu $\lim_{x \to 1} f(x) = \lim_{x \to 1} (2x - 5)$

$= 2(1) - 5$

$= -3$

$f(x=1) = 2$

$\therefore f(a) \neq \lim_{x \to a} f(x)$

$\therefore$ f(x) is not Continuous

2. Is the function $f(x) = x^2 + 3$ is Continuous if $x = 1$?

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Soln

① $f(x) = x^2 + 3$

$f(1) = 1 + 3 = 4 \in R$ is defined

② $\lim_{x \to 1} f(x) = \lim_{x \to 1} (x^2 + 3)$

$= 1 + 3 = 4$ exist

③ $f(1) = 4 = \lim_{x \to 1} f(x) = x^2 + 3 = 4$

$\therefore f(x)$ is Continuous at $x = 1$

3. $f(x) = \frac{x}{x+1}$ at $x = 3$

مجال الدالة $= R / (-1)$

① $f(3) = \frac{3}{3+1} = \frac{3}{4}$

② $\lim_{x \to 3} \frac{x}{x+1} = \frac{3}{3+1} = \frac{3}{4}$

$\therefore$ ① = ②

$\therefore$ The function is Continuous.