CV

4. If $f(x)$ is continuous or not?

$f(x) = \sqrt{x} \quad ; \quad x \geq 0$

$\lim_{x \to 0} \sqrt{x} = \sqrt{0}$ is not found

$\therefore$ The function is not continuous.

5. $f(x) = \begin{cases} \dfrac{x^2-9}{x+3} & ; \quad x \neq -3 \\ -6 & ; \quad x = -3 \end{cases}$

Solu

$\lim_{x \to -3} \dfrac{x^2-9}{x+3} = \lim_{x \to -3} \dfrac{(x-3)(x+3)}{x+3} = -6$

$f(-3) = -6$

$\therefore f(x)$ is continuous

Techniques of differentiation

① Derivative of a constant function

$f(x) = c$; c : any constant
$f'(x) = 0$

Ex
$f(x) = \sqrt{2} \Rightarrow f'(x) = 0$
$f(x) = 2\sqrt{5} + 6 \Rightarrow f'(x) = 0$

② Derivative of alinear function

$f(x) = mx + c$; m, c real no.
$f'(x) = m$

Ex
$f(x) = 7x - 2 \Rightarrow f'(x) = 7$

③ Derivative of Power functions

$f(x) = x^n$ n: integer no.
$f'(x) = n x^{n-1}$ $n = \mp 1, \mp 2, \dots$

Proof

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

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$$f'(x) = \lim_{h \to 0} \frac{[\cancel{x^n} + n x^{n-1} h + \frac{n(n-1)}{2!} x^{n-2} h^2 + \dots + n x h^{n-1} + h^n] - \cancel{x^n}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left[n x^{n-1} + \frac{n(n-1)}{2!} x^{n-2} h + \dots + n x h^{n-2} + h^{n-1}\right]}{\cancel{h}}$$

$$\therefore f'(x) = n x^{n-1}$$

Ex

1. $f(x) = x^3 \Rightarrow f'(x) = 3x^2$

2. $f(x) = x^{\frac{1}{2}} = \sqrt{x} \Rightarrow f'(x) = \frac{1}{2} x^{\frac{1}{2}-1}$

$$= \frac{1}{2\sqrt{2}}$$

3. $f(x) = \sqrt[3]{x^2} = x^{\frac{2}{3}}$

$$f'(x) = \frac{2}{3} x^{\frac{2}{3}-1}$$

$$= \frac{2}{3} x^{-\frac{1}{3}} \Rightarrow f'(x) = \frac{2}{3\sqrt[3]{x}}$$

4. $f(x) = x^8 + 12x^5 - 4x^4 + 10x^3 - 6x + 5$

$$f'(x) = 8x^7 + 60x^4 - 16x^3 + 30x^2 - 6$$