④ The derivative of the product of two functions

$$D_x(f(x)\cdot g(x)) = f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

proof

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$

Then :-

$$(f(x)\cdot g'(x))' = \lim_{h\to 0} \frac{f(x+h)\cdot g(x+h) - f(x)\cdot g(x)}{h}$$

we subtract and add the term $f(x+h)\cdot g(x)$ in the numerator :

$$= \lim_{h\to 0} \frac{f(x+h)\cdot g(x+h) + f(x+h)\cdot g(x) - f(x+h)\cdot g(x) - f(x)\cdot g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\cdot g(x+h) - f(x+h)\, g(x) - f(x+h)\, g(x) - f(x)\, g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\,[\,g(x+h)-f(x+h)\,] + g(x)\,[\,f(x+h)-f(x)\,]}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)[g(x+h)-f(x+h)]}{h} + \lim_{h\to 0} \frac{g(x)[f(x+h)-f(x)]}{h}$$

$$= \lim_{h\to 0} f(x+h)\cdot \lim_{h\to 0} \frac{g(x+h)-f(x+h)}{h} + \lim_{h\to 0} g(x) \lim \frac{f(x+h)-f(x)}{h}$$

$$= f(x)\; g'(x) + g(x)\cdot f'(x)$$

1. **Ex** $f(x) = (x^3+1)(2x^2+8x-5)$

Find $f'(x)$?

Solu $f'(x) = (x^3+1)(4x+8) + (2x^2+8x-5)(3x^2)$

2. $f(x) = (3x^4 - 5x)(x^2 + \sqrt{x})$

$f'(x) = (12x^3 - 5)(x^2 + \sqrt{x}) + (3x^4 - 5x)\left(2x + \frac{1}{2\sqrt{x}}\right)$

3. $f(x) = \frac{1+3x}{3x}(3-x)$

$$f'(x) = \left(\frac{1+3x}{3x}\right)(-1) + (3-x)\left(\frac{3x \cdot 3 - (1+3x) \cdot 3}{(3x)^2}\right)$$

$$= \frac{-1-3x}{3x} + (3-x) \cdot \frac{\cancel{9x} - 3 - \cancel{9x}}{9x^2}$$

$$= \frac{-1-3x}{3x} + (3-x) \cdot \frac{-\cancel{3}}{{}_3\cancel{9}x^2}$$

$$= \frac{-1-3x}{3x} + (3-x) \cdot \frac{-1}{3x^2}$$

$$= \frac{-1-3x}{3x} + \frac{(-3+x)}{3x^2}$$

$$= \frac{-1}{3x} - \frac{3x}{3x} \mp \frac{3}{3x^2} + \frac{x}{3x^2}$$

$$= \cancel{\frac{-1}{3x}} - 1 \mp \frac{1}{x^2} + \cancel{\frac{1}{3x}} = -1 - \frac{1}{x^2}$$

(5) The derivative of the Qhotient of two functions is :-

H.w.

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{(g(x))^2}$$

Ex Find the derivative of the following:-

1. $f(x) = \dfrac{3x^2 - x + 2}{4x^2+5}$

$$f'(x) = \frac{(4x^2+5)(6x-1) - (3x^2-x+2)(8x)}{(4x^2+5)^2}$$

$$= \frac{24x^3 - 4x^2 + 30x - 5 - 24x^3 + 8x^2 - 16}{16x^4 + 40x^2 + 25}$$

2. $y = x^2 + \tan x^3$

$$y' = 2x + \sec^2 x^3 (3x^2)$$

$$= 2x + 3x^2 \sec^2 x^3$$

3. $y = e^x (x^2+1)$

$$y' = e^x (2x) + (x^2+1) e^x \cdot 1$$

$$= 2xe^x + e^x (x^2+1)$$

4. $y = \dfrac{\sec x^2}{x}$

$$y' = \frac{x \sec x^2 \tan x^2 (2x) - \sec x^2 (1)}{x^2}$$