Ex Find the derivative of the function :-

$f(x) = \frac{7}{\sqrt{x}}$ using the diffinition

Solu

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = \frac{7}{\sqrt{x}} \quad ; \quad f(x+h) = \frac{7}{\sqrt{x+h}}$$

$$\therefore f'(x) = \lim_{h \to 0} \frac{\frac{7}{\sqrt{x+h}} - \frac{7}{\sqrt{x}}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{7\sqrt{x} - 7\sqrt{x+h}}{\sqrt{x+h}\,\sqrt{x}}}{h}$$ multiple Conjugate

$$= \lim_{h \to 0} \frac{\frac{7\sqrt{x} - 7\sqrt{x+h}}{\sqrt{x+h}\,\sqrt{x}} \cdot \frac{7\sqrt{x} + 7\sqrt{x+h}}{7\sqrt{x} + 7\sqrt{x+h}}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{49x + \cancel{49\sqrt{x}\sqrt{x+h}} - \cancel{49\sqrt{x}\sqrt{x+h}} - 49(x+h)}{\sqrt{x+h}\,\sqrt{x}\,(7\sqrt{x} + 7\sqrt{x+h})}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{49x - 49(x+h)}{7x\sqrt{x+h} + 7(x+h)\sqrt{x}}}{h}$$

$$= \lim_{h \to 0} \frac{7(\cancel{7x} - \cancel{7x} - 7h)}{7x\sqrt{x+h} + 7\sqrt{x}(x+h)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{7\,\cancel{-49}\,\cancel{h}}{\cancel{7}[x\sqrt{x+h} + \sqrt{x}(x+h)} \cdot \frac{1}{\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-7}{x\sqrt{x+h} + (x+h)\sqrt{x}}$$

$$\therefore f'(x) = \frac{-7}{x\sqrt{x} + \sqrt{x}\ x}$$

$$= \frac{-7}{2x\sqrt{x}}$$

or

$$f(x) = \frac{7}{\sqrt{x}}$$

$$f'(x) = \frac{\sqrt{x}\,(0) - 7\ \frac{1}{2} x^{-\frac{1}{2}}}{(\sqrt{x})^2}$$

$$= \frac{-7}{2x\sqrt{x}}$$

Ex $f(x) = \sqrt{x}$

Solu $f(x+h) = \sqrt{x+h}$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$