Implicit Differentiation

Def.: We will say that a given equation in x and y defines the function f implicitly y if the graph of $y=f(x)$ coincides with a portion of the graph of the equation

Ex Use implicit differentiation to find $\frac{dy}{dx}$ if:-

1. $x^2+y^2=1$

Solu $2x+2y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{-x}{y}$

2. $y^2-5xy=6$

$$2y\frac{dy}{dx}-5\left[x\frac{dy}{dx}+y\cdot 1\right]=0$$

$$2y\frac{dy}{dx}-5x\frac{dy}{dx}-5y=0$$

$$\frac{dy}{dx}\left[2y-5x\right]=+5y \Rightarrow \frac{dy}{dx}=\frac{5y}{2y-5x}$$

3. $5y^2+\sin y=x^2$

$$10y\frac{dy}{dx}+\cos y\frac{dy}{dx}=2x$$

$$\frac{dy}{dx}(10y+\cos y)=2x$$

$$\frac{dy}{dx}=\frac{2x}{10y+\cos y}$$

Ex 5. $x^2+y^2+2xy=4$

Solu $2x+2y\frac{dy}{dx}+2[x\frac{dy}{dx}+y\cdot 1]=0$

$2x+2y\frac{dy}{dx}+2x\frac{dy}{dx}+2y=0$

$\frac{dy}{dx}[2y+2x]=-2x-2y$

$\frac{dy}{dx}=\frac{-2(x+y)}{2(y+x)}$

$\frac{dy}{dx}=-1$

6. $y=2x+\sin(y-2x)$

Solu $\frac{dy}{dx}=2+\cos(y-2x)\cdot(\frac{dy}{dx}-2)$

$\frac{dy}{dx}=2+\cos(y-2x)\frac{dy}{dx}-2\cos(y-2x)$

$2\cos(y-2x)-2=\cos(y-2x)\frac{dy}{dx}-\frac{dy}{dx}$

$2[\cos(y-2x)-1]=\frac{dy}{dx}(\cos(y-2x)-1)$

$\frac{dy}{dx}=\frac{2(\cos(y-2x)-1)}{(\cos(y-2x)-1)}$

$\frac{dy}{dx}=2$

The Chain Rule

Theorem :- If g is differentiable at x and f is differentiable at $g(x)$, then the compsition $(f \circ g)$ is differentiable at x, moreover :-

$$\cancel{(f \circ g)'(x) = f'(g(x) \cdot g'(x))}$$

or $y = f(g(x))$ and $u = g(x)$, then

$y = f(u)$ and :-

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Ex 1. If $y = \sin^2(3x^5 + x^4 - 2)$ Find $\frac{dy}{dx}$

Solu

let $u = g(x) = 3x^5 + x^4 - 2$ and $f(u) = f(g(x)) = \sin^2 u$

$$\therefore \frac{dy}{du} = 2 \sin u \cdot \cos u = \sin 2u$$

$$\frac{du}{dx} = 15x^4 + 4x^3$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \sin 2u \cdot (15x^4 + 4x^3)$$

$$= [\sin 2(3x^5 + x^4 - 2)](15x^4 + 4x^3)$$