4. Integration using Completing the Square method

This method is used when there is a quadratic equation that cannot be solve by fractional division, as follows:-

Ex

1. $\int \frac{1}{x^2+2x+2} dx$

We take the square of half the coefficient of x, that is:- $\mp(\frac{1}{2}x)^2$

$$= \int \frac{dx}{x^2+2x+2+1-1}$$

$$= \int \frac{dx}{x^2+2x+1+(2-1)}$$

$$= \int \frac{dx}{(x+1)^2+1} = \tan^{-1}(x+1)+c$$

or

let $u = x+1 \Rightarrow du = dx$

$$= \int \frac{du}{u^2+1} = \tan^{-1}u + c = \tan^{-1}(x+1)+c$$

2. $\int \frac{dx}{x-2\sqrt{(x^2-4x+3}}$ ∓ 4

Solu

$= \int \frac{dx}{x-2\sqrt{x^2-4x+3\pm 4-4}}$

$= \int \frac{dx}{x-2\sqrt{(x-2)^2+(3-4)}}$

$= \int \frac{dx}{x-2\sqrt{(x-2)^2-1}} = \sec^{-1}(x-2)+c$

or let $u=x-2 \Rightarrow du=dx$

$= \int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1}u+c = \sec^{-1}(x-2)+c$

3. $\int \frac{x-4}{x^2-6x+10}dx$ ∓ 9

Solu $= \int \frac{x-4}{x^2-6x+10+9-9}dx = \int \frac{x-4}{x^2-6x+9+(10-9)}dx$

$= \int \frac{x-4}{(x-3)^2+1}dx$

let $u=x-3 \Rightarrow du=dx \Rightarrow x=u+3$

$$= \int \frac{u+3-4}{u^2+1}\,du = \int \frac{u-1}{u^2+1}\,du$$

$$= \int \frac{u}{u^2+1}\,du - \int \frac{1}{u^2+1}\,du$$

$$= \frac{1}{2}\ln|u^2+1| - \tan^{-1}u + c$$

$$= \frac{1}{2}\ln|(x-3)^2+1| - \tan^{-1}(x-3) + c$$

3. $\int \frac{dx}{20+8x-x^2}$ ∓ 16

Sol/

$$= \int \frac{dx}{\sqrt{-(x^2-8x-20)}} = \int \frac{dx}{\sqrt{-(x^2-8x-20+16-16}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-8x+6)+36}} = \int \frac{dx}{\sqrt{-(x-4)^2+36}}$$

$$= \int \frac{dx}{\sqrt{36-(x-4)^2}} = \int \frac{dx}{6\sqrt{1-(\frac{x-4}{6})^2}}$$

$$= \sin^{-1}\left(\frac{x-4}{6}\right) + c$$