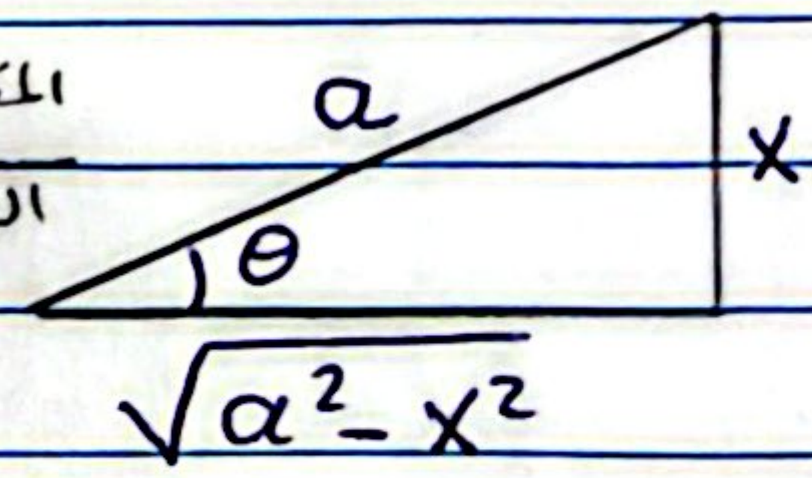
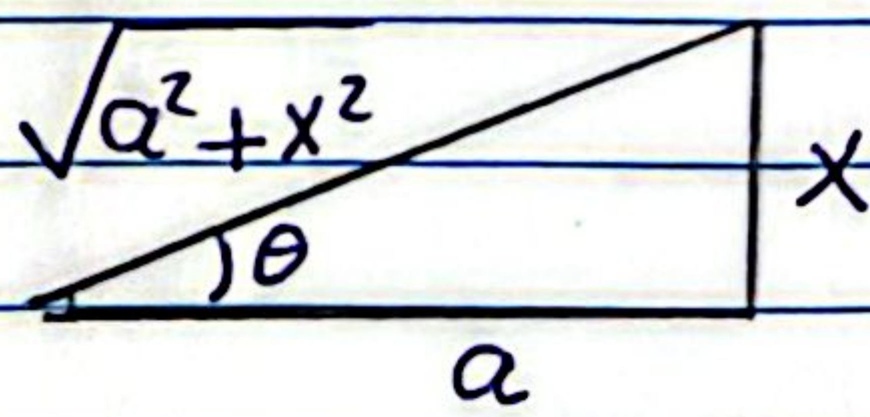
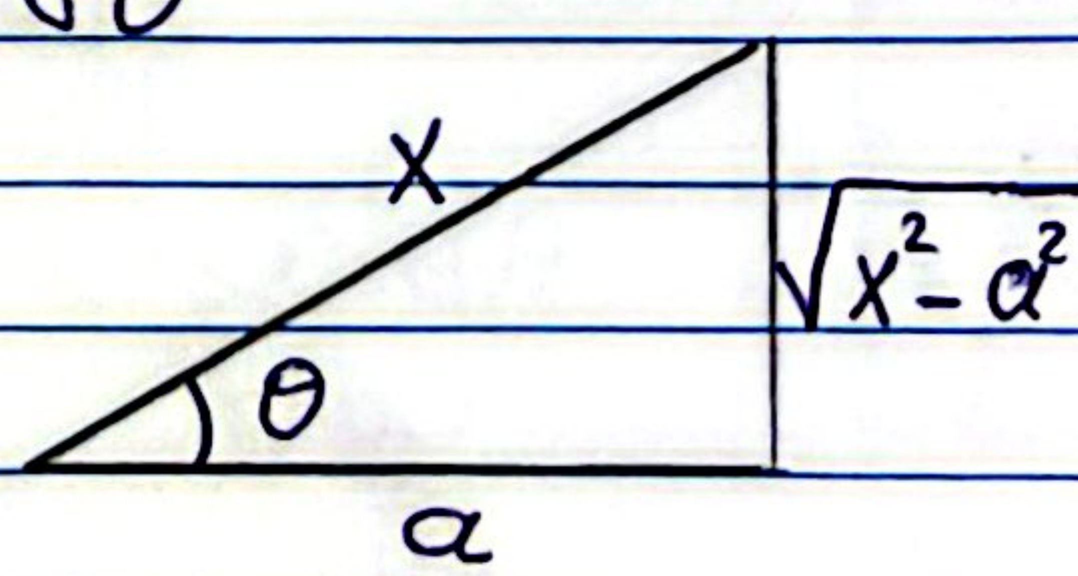
5) Integration by Substitution with trigonometric Functions:-

Sometimes the function to be integrated can be converted into a trigonometric function that can be integrated في بعض الأحيان يمكن تحويل الدالة المراد تكاملها إلى دالة مثلثية يمكن تكاملها
This integration is called trigonometric substitution
This method is used if the integration contains:- الاستبدال المثلثي

$\sqrt{a^2-x^2}$ or $\sqrt{a^2+x^2}$ or $\sqrt{x^2-a^2}$

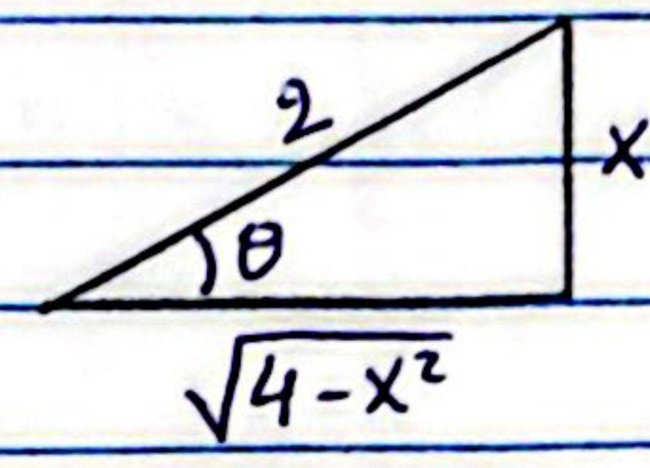
To find this integration, we use substitutions as shown in the following table:-

The amount	التعويض Compensation	الشكل Shape
$\sqrt{a^2-x^2}$ $\sin^2\theta+\cos^2\theta=1$	$x=a\sin\theta$ $dx=a\cos\theta\,d\theta$ $\frac{\text{المقابل}}{\text{الوتر}}$	
$\sqrt{a^2+x^2}$ $\sec^2\theta=\tan^2\theta+1$	$x=a\tan\theta$ $dx=a\sec^2\theta\,d\theta$ $\frac{\text{المقابل}}{\text{المجاور}}$	
$\sqrt{x^2-a^2}$ $\tan^2\theta=1-\sec^2\theta$	$x=a\sec\theta$ $dx=a\sec\theta\tan\theta\,d\theta$ $\frac{\text{الوتر}}{\text{المجاور}}$	

Ex Find the following integrals :-

1. $\int \frac{dx}{\sqrt{4-x^2}}$

Sol^n let $x = 2\sin\theta \Rightarrow dx = 2\cos\theta \, d\theta$



$$\therefore \int \frac{dx}{\sqrt{4-x^2}} = \int \frac{2\cos\theta \, d\theta}{\sqrt{4-4\sin^2\theta}}$$

$$= \int \frac{2\cos\theta \, d\theta}{\sqrt{4(\underbrace{1-\sin^2\theta}_{\cos^2 x})}} = \int \frac{2\cos\theta \, d\theta}{2\cos\theta}$$

$$= \int d\theta = \theta + C$$

$$x = 2\sin\theta \Rightarrow \sin\theta = \frac{x}{2} \Rightarrow \theta = \sin^{-1}\frac{x}{2}$$

$$\therefore \int \frac{dx}{\sqrt{4-x^2}} = \sin^{-1}\frac{x}{2} + C$$

2. $\int \sqrt{4-x^2} \, dx$

Sol^n let $x = 2\sin\theta \Rightarrow dx = 2\cos\theta \, d\theta$

$$\therefore \int \sqrt{4-x^2} \, dx = \int \sqrt{4-(2\sin\theta)^2} \; 2\cos\theta \, d\theta$$