Integration of trigonometric Functions :-

a) Integration of odd Functions

We have : $\sin^2 x + \cos^2 x = 1$, Then:

$$\sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$

Therefore, the integral of functions containing odd exponents of Sin and cos is as follows:-

$$\int \sin^m x \cos^n x \, dx$$

where : m is odd and n is even

Then the integration of these functions is as Follows:-

$$\int \sin^m x \cos^n x \, dx = \int \sin^m x \cos x \cos^{m-1} x \, dx$$

Ex

1. $\int \sin^3 x \cos^2 x \, dx$

Solu

$= \int \sin x \sin^2 x \cos^2 x \, dx$

$= \int \sin x (1 - \cos^2 x) \cos^2 x \, dx$

$= \int \sin x \cos^2 x - \sin^x \cos^4 x \, dx = \frac{-\cos^3 x}{3} + \frac{\cos^5 x}{5} + c$

let $u = \cos x \Rightarrow du = -\sin x \, dx$

$= -\int (1 - u^2) u^2 \, du$

$= -\int (u^2 - u^4) \, du$

$= -\frac{u^3}{3} + \frac{u^5}{3} + c$

2. $\int \cos^5 x \sin^2 x \, dx$

Solu

$= \int \cos x \cos^4 x \sin^2 x \, dx$

$= \int \cos x (\cos^2 x)^2 \sin^2 x \, dx$

$= \int \cos x (1 - \sin^2 x)^2 \sin^2 x \, dx$

$= \int \cos x (1 - 2\sin^2 x + \sin^4 x) \sin^2 x \, dx$

$= \int (\cos x \sin^2 x - 2\cos x \sin^4 x + \cos x \sin^5 x) \, dx$

$= \frac{\sin^3 x}{3} - 2\frac{\sin^5 x}{5} + \frac{\sin^7 x}{7} + c$