b) Integration of even functions

The following relationships are used:

① $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

② $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

Ex 1. $\int \sin^2 x \, dx$

$= \int \frac{1}{2}(1 - \cos 2x) dx = \frac{1}{2}\left[\int dx - \int \cos 2x \, dx\right]$

$= \frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{2}\sin 2x$

$= \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

2. $\int \sin x \cos^3 x \, dx = \int \sin x \cos x \cos^2 x \, dx$

$= \int \sin x \cos x (1 - \sin^2 x) dx$ → OR Let $u = \cos x$, $du = -\sin x \, dx$

$= \int (\sin x \cos x - \cos x \sin^3 x) dx$ $= \int u(1 - u^2) du$

$= -\frac{\sin^2 x}{2} + \frac{\sin^4 x}{4} + C$ $= \int (u - u^3) du$

$= -\frac{u^2}{2} + \frac{u^4}{4}$

$= -\frac{\cos^2 x}{2} + \frac{\sin^4 x}{4} + C$

H.W. $\int \sin^4 x \, dx$; $\int \cos^2 x \, dx$; $\int \sin^6 x \, dx$, $\int \cos^6 x \, dx$

Power Series

Sequence : It is a function whose starting point is the set of positive Integers, for example :-

$a(n)$; $a(1)=1$; $a(2)=2$; $a(3)=3$ and so on

n	a(n)
0	0
1	1
2	2
⋮	⋮

Ex

1. $a(n) = \frac{1}{n}$; $a(0)$ = not defined

$a(1) = 1$

$a(2) = \frac{1}{2}$

$a(3) = \frac{1}{3}$

⋮

Ex

2. $a(n) = \frac{\ln(n)}{n^2}$

$a(1) = \frac{\ln(1)}{1} = 0$

$a(2) = \frac{\ln(2)}{4} = 0.1732$

$a(3) = \frac{\ln(3)}{9} = 0.1220$

For each squence the term $a(n)$ is called the n-th term اكد التوني