

Ex Find the derivative  $y'$  for the following functions:

1.  $y = x^2 \tan x$

$$y' = x^2 \cdot \sec^2 x + \tan x \cdot 2x$$

$$= x^2 \sec^2 x + 2x \tan x \quad /$$

2.  $y = \sin(x^2 + 3)$

$$y' = \cos(x^2 + 3) \cdot 2x \quad /$$

$$= 2x \cos(x^2 + 3)$$

3.  $y = 2 \cos x - 3 \sin x$

$$y' = -2 \sin x - 3 \cos x$$

4.  $y = \frac{\sin x}{1 + \cos x}$

$$y' = \frac{(1 + \cos x)(\cos x) - \sin x(-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2} = \frac{\cos x + 1}{(1 + \cos x)^2} = \frac{1}{1 + \cos x}$$

5.  $y = \sec x \tan x$

$$y' = \sec x \cdot (\sec^2 x) + \tan x \cdot (\sec x \tan x)$$

$$= \sec^3 x + \sec x \tan^2 x = \sec x (\sec^2 x + \tan^2 x)$$

Ex Prove that  $\frac{d}{dx} \tan x = \sec^2 x$

Soly

$$\frac{d}{dx} \tan x = \frac{d}{dx} \left[ \frac{\sin x}{\cos x} \right]$$

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$$= \frac{\cos x (\cos x) - \sin x (-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

H.w.

Prove that :-

$$1. \frac{d}{dx} \sec x = \sec x \tan x$$

$$2. \frac{d}{dx} \cot x = -\csc^2 x$$

$$3. \frac{d}{dx} \cos x = -\sin x$$

Theorem

$$1. \sin^2 x + \cos^2 x = 1$$

$$2. \sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$$

$$3. \sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha$$

$$4. \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$5. \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$6. \sin(2\alpha) = 2 \sin \alpha \cos \alpha$$

$$7. \cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha$$

$$8. \sec^2 \alpha = 1 + \tan^2 \alpha$$

Ex Find  $y''(\frac{\pi}{4})$  if  $f(x) = \sec x$

$$y = \sec x$$

$$y' = \sec x \tan x$$

$$y'' = \sec x \sec^2 x + \tan x \cdot \sec x \tan x$$

$$= \sec^3 x + \tan^2 x \sec x$$

$$y''(\frac{\pi}{4}) = \sec^3(\frac{\pi}{4}) + \tan^2(\frac{\pi}{4}) \sec(\frac{\pi}{4})$$

$$= \sec^3(45) + \tan^2(45) \sec(45)$$

$$= (\sqrt{2})^3 + (1) \sqrt{2} = 3\sqrt{2}$$

Ex

Evaluate the following integrals:-

$$1. \int \sin(x+y) dx = -\cos(x+y) + c \quad \checkmark$$

$$2. \int \cos 5x dx = \frac{1}{5} \sin 5x + c \quad \checkmark$$

$$3. \int (\frac{1}{x^2} + \sec^2 x) dx = \int \frac{1}{x^2} dx + \int \sec^2 x dx \quad \checkmark$$

$$= -\frac{1}{x} + \tan x + c$$

$$4. \int \sin^2 x dx = \int \frac{1 - \cos 2x}{2} dx = \frac{1}{2} \int dx - \int \cos 2x dx$$

$$= \frac{1}{2} x - \frac{1}{4} \sin 2x + c$$