The Inverse Trigonometric Functions

1- $y = \sin^{-1} x$

The inverse sin function, is defined to be the inverse of the restricted sine function

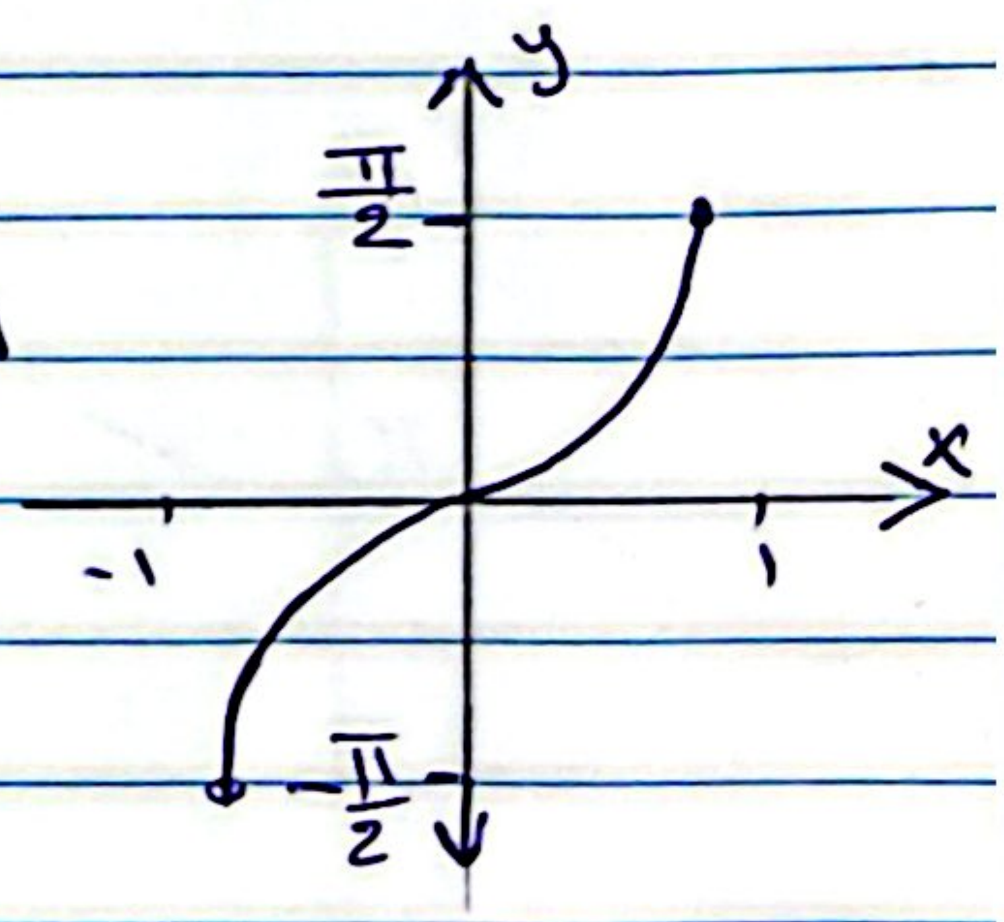
$\sin x \quad , -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$D_y = [-1, 1]$ or $D_y = \{x : x \in R , -1 \leq x \leq 1\}$

$R_y = [-\frac{\pi}{2}, \frac{\pi}{2}]$ or $R_y = \{y : y \in R ; -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\}$

$\because \sin^{-1}(\sin x) = x \quad ; \text{if } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$\sin(\sin^{-1} x) = x \quad ; \text{if } -1 \leq x \leq 1$



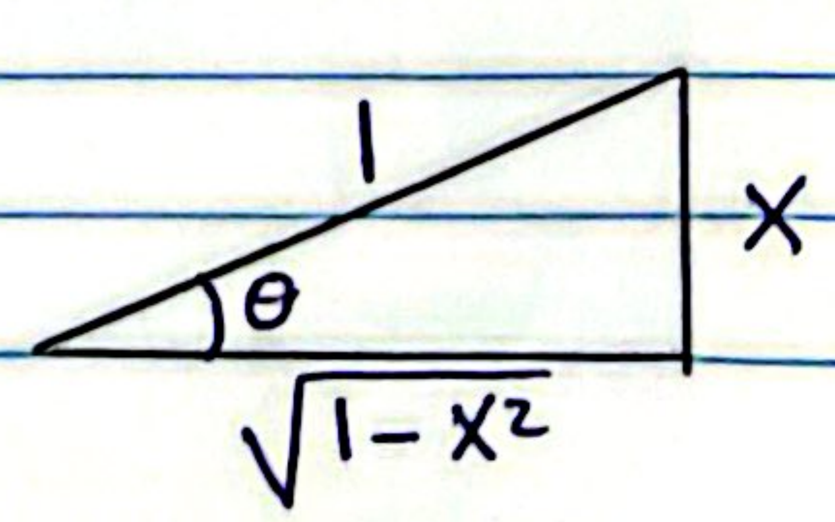
$\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx} \quad ; -1 < u < 1$

$\int \frac{1}{\sqrt{1-u^2}} du = \sin^{-1} u + c$

Note:- $\frac{1}{\sin x} \neq \sin^{-1} x$ دالة مثلثية

But

$(\sin x)^{-1} \neq \sin^{-1} x$ دالة مثلثية عكسية



2- $y = \cos^{-1} x$.

The inverse cosine function, is defined to be the inverse of the restricted cosine function

$\cos x \quad ; \quad 0 \le x \le \pi$

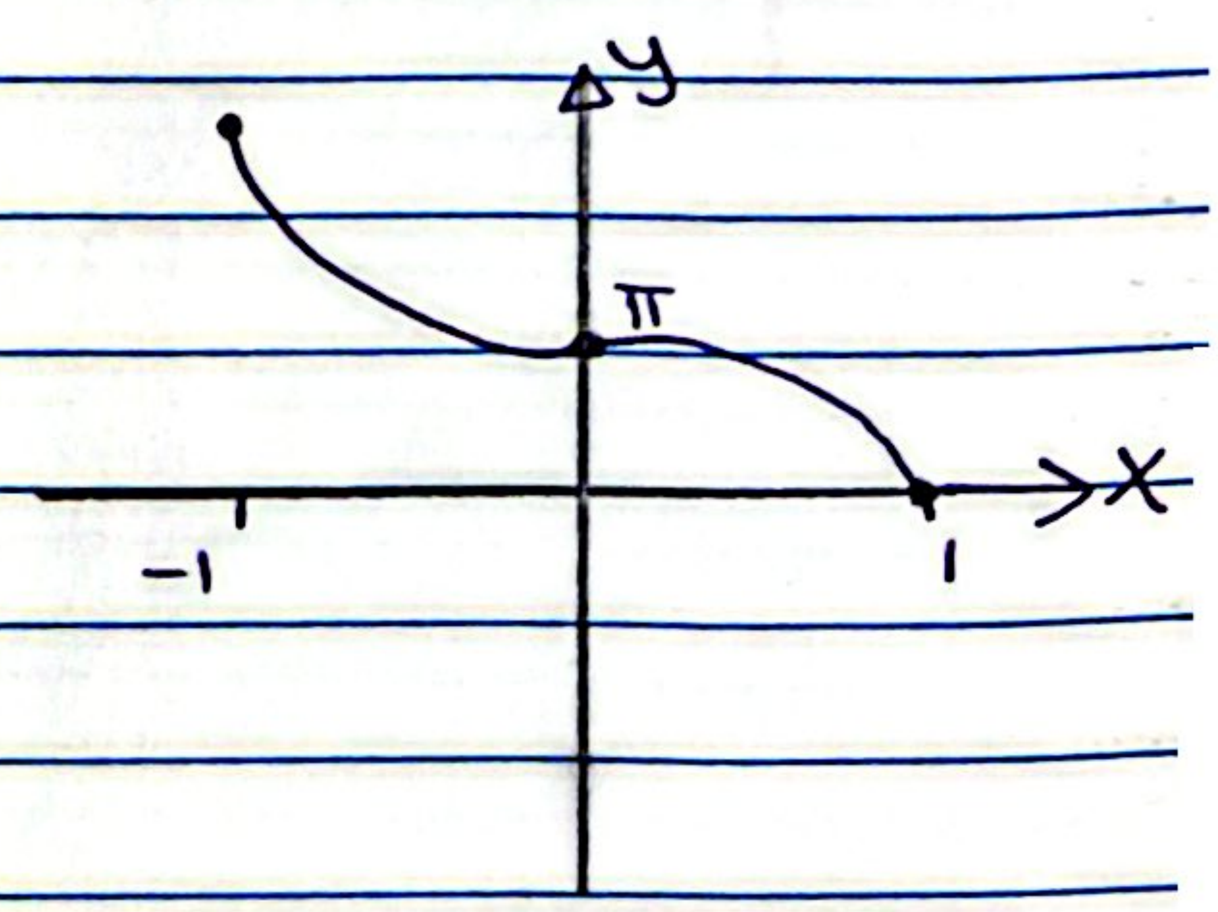
$D_y = [-1, 1] \quad ; \quad R_y = [0, \pi]$

$\cos^{-1}(\cos x) = x \quad ; \text{if } 0 \le x \le \pi$

$\cos(\cos^{-1} x) = x \quad \text{if } -1 \le x \le 1$

$$\frac{d}{dx}(\cos^{-1} u) = \frac{-1}{\sqrt{1-u^2}} \frac{du}{dx} \quad ; -1 < u < 1$$

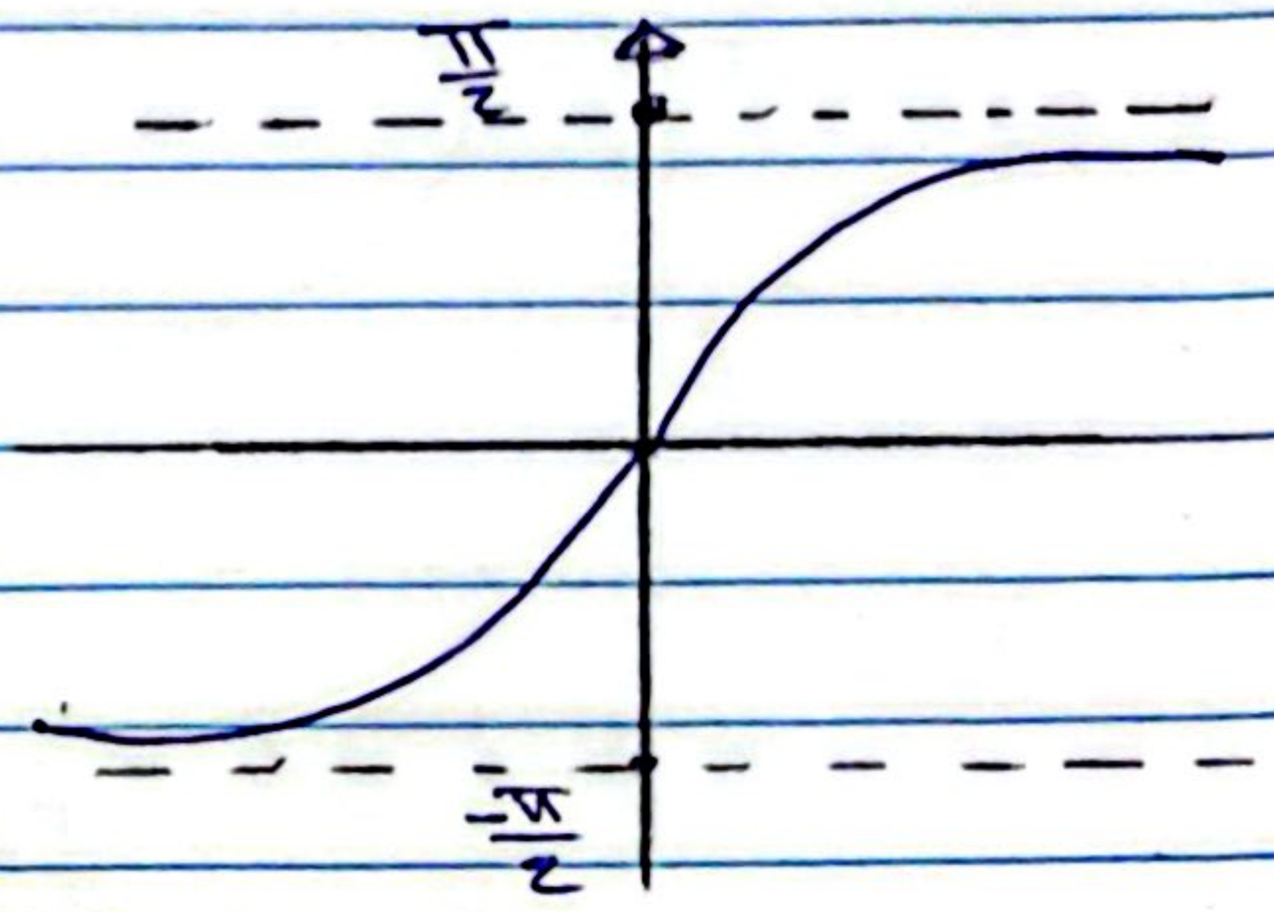
$$\int \frac{du}{\sqrt{1-u^2}} = -\cos^{-1} u + C$$



3- $y = \tan^{-1} x$

The inverse tangent function, is defined to be the inverse of the restricted tangent function

$\tan x \quad ; \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$

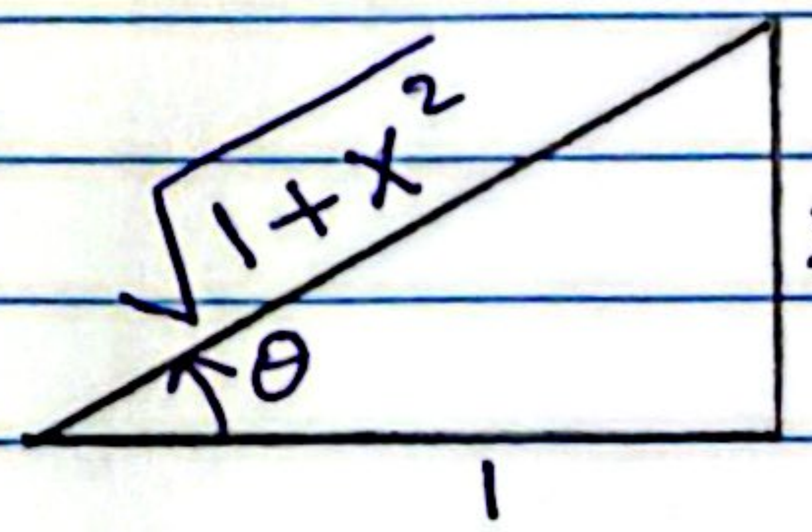


$D_y = (-\infty, \infty) \quad ; \quad R_y = (-\frac{\pi}{2}, \frac{\pi}{2})$

$\tan(\tan^{-1} x) = x \quad \text{if} \quad -\infty < x < \infty$

$\tan^{-1}(\tan x) = x \quad \text{if} \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$

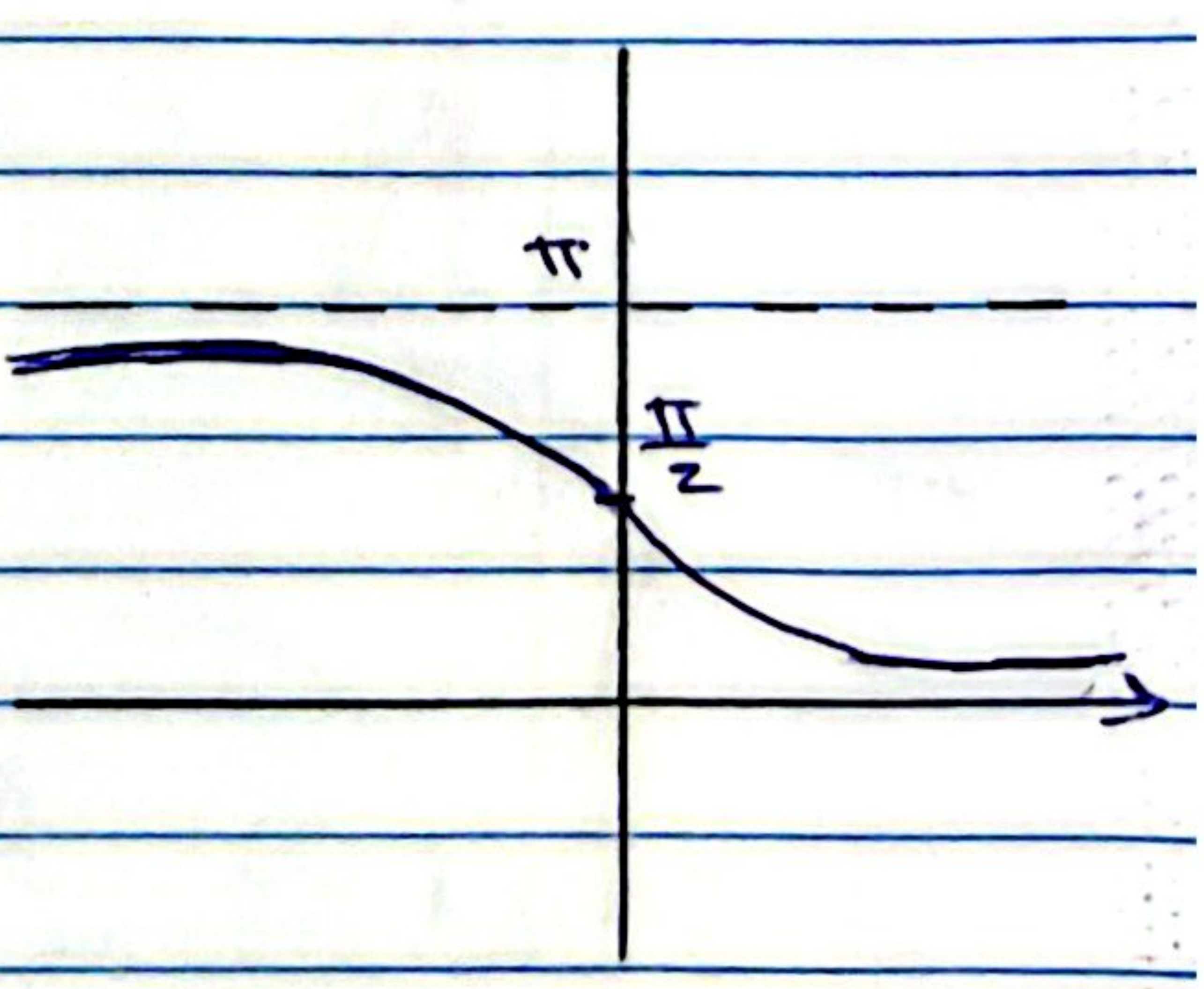
$$\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx} \quad ; \quad (-\infty < u < \infty)$$



$$\int \frac{1}{1+u^2} du = \tan^{-1} u + c$$

4- $y = \cot^{-1} x$

$\cot x \quad ; \quad 0 < x < \pi$



$D_y = (-\infty, \infty) \quad ; \quad R_y = (0, \pi)$

$\cot^{-1}(\cot x) = x \quad \text{if} \quad 0 < x < \pi$

$\cot(\cot^{-1} x) = x \quad \text{if} \quad -\infty < x < \infty$

$$\frac{d}{dx}(\cot^{-1} u) = \frac{-1}{1+u^2} \cdot \frac{du}{dx} \quad ; \quad -\infty < u < \infty$$

$$\int \frac{du}{1+u^2} = -\cot^{-1} u + c$$