2) Integration by Parts التجزئة

Let u and v any two function of x, then:-

$$\frac{\partial}{\partial x}(u \cdot v) = u\frac{dv}{dx} + v\frac{\partial u}{dx}$$

By integrating both sides we get:-

$$\int \frac{\partial}{\partial x}(u \cdot v)dx = \int u\frac{dv}{\partial x}\,dx + \int v\frac{du}{dx}\,dx$$

$$u \cdot v = \int u\,dv + \int v\,du$$

$$\therefore \int u\,dv = u \cdot v - \int v\,du$$

Ex

Evalute the Following integrals:-

1. $\int x \sin x\,dx$

let $u = x \Rightarrow du = dx$

$dv = \sin x\,dx \Rightarrow v = -\cos x$

$$\therefore \int x \sin x\,dx = -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + C$$

2- $\int \tan^{-1} dx$

let $u = \tan^{-1} x \Rightarrow du = \frac{1}{1+x^2} dx$

$dv = dx \Rightarrow v = x$

$\therefore \int \tan^{-1} dx = x \tan^{-1} x - \int x \frac{1}{1+x^2} dx$

$= x \tan^{-1} x - \frac{1}{2} \ln|1+x^2| + C$

$= x \tan^{-1} x + \ln \frac{1}{\sqrt{1+x^2}} + C$

3- $\int e^x \sin x \, dx$

let $u = e^x \Rightarrow du = e^x dx$

$dv = \sin x \, dx \Rightarrow v = -\cos x$

$\therefore \int e^x \sin x dx = -e^x \cos x + \int e^x \cos x \, dx$

↓

let $u_1 = e^x \Rightarrow du = e^x dx$

$dv_1 = \cos x dx \Rightarrow v = \sin x$

$\therefore \int e^x \sin x dx = -e^x \cos x + [e^x \sin x - \int e^x \sin x \, dx]$

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