

3) Integration of Fractions

Let f and g be two functions defined by:

$$R(x) = \frac{f(x)}{g(x)} \quad ; \quad g(x) \neq 0$$

Such that $f(x)$ and $g(x)$ are multiple polynomials, then two cases:-

A) Case one

When the order of the numerator is less than the order of the denominator; that is, the function is proper.

B) Case two

When the order of the numerator is greater or equal than the order of the denominator; that is, the function is improper.

For examples of types of functions:-

$$1. \quad f(x) = \frac{x+1}{x^3-3x} \quad \text{proper} \quad \text{سليم}$$

$$2. \quad f(x) = \frac{x^3+5x^2}{x^2-2} \quad \text{improper} \quad \text{غير سليم}$$

$$3. \quad f(x) = \frac{x+1}{x-2} \quad \text{improper}$$

$$4. \quad f(x) = \frac{x}{x^2+2} \quad \text{proper}$$

Improper Function :-

If the order of the numerator is greater than or equal to the order of the denominator, that is the function is improper :-

For example :-

$$1. \int \frac{x+1}{x-2} dx$$

To find the integral we use long division as follows :-

$$\begin{array}{r} 1 \\ x-2 \overline{) x+1} \\ \underline{-x+2} \\ 3 \end{array}$$

$$\therefore \int \frac{x+1}{x-2} dx = \int 1 dx + \int \frac{3}{x-2} dx$$

$$= x + 3 \ln|x-2| + c$$

$$2. \int \frac{x^3+5x^2}{x^2+2} dx$$

$$\begin{array}{r} x+5 \\ x^2+2 \overline{) x^3+5x^2} \\ \underline{-x^3+2x} \\ 5x^2-2x \end{array}$$

$$\therefore \int \frac{x^3+5x^2}{x^2+2} dx = \int (x+5) dx + \int \frac{-2x-10}{x^2+2} dx$$

$$= \frac{x^2}{2} + 5x - \int \frac{2x}{x^2+2} dx - \int \frac{10}{x^2+2} dx$$

$$\begin{array}{r} 5x^2+10 \\ \underline{-2x-10} \end{array}$$

$$= \frac{x^2}{2} + 5x - \ln|x^2+2| - \frac{10}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + c$$