Case two :- Proper function

If the order of the numerator is less than the order of the denominator, then the function is proper, there are several cases:-

A) If the denominator decomposes into real and different roots the..

1. Factoring the denominator into prime factors:-

$(x+a), (x+b), \dots, (x+d)$

the integration becomes to:

$$= \int \frac{A}{x+a} dx + \int \frac{B}{x+b} dx \dots +$$

2. Mutiply both sides of expression by the denominator of the fraction to be divided to get rid of fractions:- نقسم للتخلص من المقام

3. Solve equation
4. perform integration

Ex: Find the following integral

1. $\int \frac{1}{x^2-4} dx$

$$\int \frac{1}{x^2-4} dx = \int \frac{A}{x+2} dx + \int \frac{B}{x-2} dx$$

$$\left[\frac{1}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2} \right] (x^2-4) \text{ or } (x-2)(x+2)$$

$$1 = A(X-2) + B(X+2)$$
$$1 = AX - 2A + BX + 2B$$

$$\therefore 1 = 2B - 2A \quad (1)$$

$$0 = AX + BX \quad (2) \quad \div X \Rightarrow A = -B$$

Substituting the value of A into equation 1 we get :-

$$1 = 2B - 2(-B) \Rightarrow B = \frac{1}{4} \text{ and } A = -\frac{1}{4}$$

By substituting the values A and B in the main equation, we get the result of the integration as follows :-

$$\int \frac{1}{X^2-4} dx = \int \frac{-\frac{1}{4}}{X-2} dx + \int \frac{\frac{1}{4}}{X+2} dx$$

$$= -\frac{1}{4} \ln|X-2| + \frac{1}{4} \ln|X+2| + C$$

2. $\int \frac{dx}{X^2+5X+6}$

$$X^2+5X+6 = (X+3)(X+2)$$

$$\therefore \int \frac{1}{X^2+5X+6} dx = \int \frac{A}{X+3} dx + \int \frac{B}{X+2} dx$$

$$\left(\frac{1}{X^2+5X+6}\right) = \left(\frac{A}{X+3} + \frac{B}{X+2}\right)(X^2+5X+6) \text{ or } (X+3)(X+2)$$

$$1 = A(X+2) + B(X+3)$$
$$1 = AX + 2A + BX + 3B$$