B) If the denominator is decomposed into roots or first-order factors, some of which are repeated

denominator: $(x+a)(x+b)^2$

$(x+a)(x+b)^3$

$(x+a)^4(x+b)$

and so on

The integration is as follows:-

1. $\int \frac{A}{x+a}\,dx + \int \frac{B}{x+b}\,dx + \int \frac{C}{(x+b)^2}$

2. $\int \frac{A}{x+a}\,dx + \int \frac{B}{(x+b)}\,dx + \int \frac{C}{(x+b)^2}\,dx + \int \frac{D}{(x+b)^3}\,dx$

3. $\int \frac{A}{x+a}\,dx + \int \frac{B}{(x+a)^2}\,dx + \int \frac{C}{(x+a)^3}\,dx + \int \frac{D}{(x+a)^4}\,dx + \int \frac{E}{x+b}\,d$

Ex

Evaluate as the following:-

$$\int \frac{x}{(x-1)^2}\,dx$$

$$\int \frac{x}{(x-1)^2}\,dx = \int \frac{A}{(x-1)}\,dx + \int \frac{B}{(x-1)^2}\,dx$$

$$\left[\frac{x}{(x-1)^2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2}\right] * (x-1)^2$$

$$X = A(X-1) + B$$
$$X = AX - A + B$$

$$X = AX \Rightarrow A = 1 \quad (1)$$

$$B - A = 0 \Rightarrow B = A = 1 \quad (2)$$

$$\therefore \int \frac{X}{(X-1)^2} dx = \int \frac{1}{X-1} dx + \int \frac{1}{(X-1)^2} dx$$

$$= \ln|X-1| + \int (X-1)^{-2} dx$$

$$= \ln|X-1| + \frac{(X-1)^{-1}}{-1}$$

$$= \ln|X-1| - \frac{1}{(X-1)} + c$$

2. $$\int \frac{X+2}{(X-1)(X-2)^2} dx$$

$$\int \frac{X+2}{(X-1)(X-2)^2} dx = \int \frac{A}{X-1} dx + \int \frac{B}{X-2} dx + \int \frac{C}{(X-2)^2} dx$$

$$\left[\frac{X+2}{(X-1)(X-2)^2} = \frac{A}{X-1} + \frac{B}{X-2} + \frac{C}{(X-2)^2}\right] * (X-1)(X-2)^2$$

$$X+2 = A(X-2)^2 + B(X-1)(X-2) + C(X-1)$$
$$X+2 = AX^2 - 4AX + 4A + BX^2 - 3BX + 2B + CX - C$$

$$AX^2 + BX^2 = 0 \quad (1) \quad \div X^2 \Rightarrow A = -B$$
$$-4AX - 3BX + CX = X \quad (2) \quad \div X \Rightarrow -4A - 3B + C = 1$$
$$4A + 2B - C = 2 \quad (3)$$