

c) If some roots are of the second degree and not analytical, then the numerator is straight linear equation:

$$\int \frac{Ax+B}{x^2+a}$$

For example

$$1. \int \frac{x^2+2x}{x(x^2+1)} dx$$

$$\int \frac{x^2+2x}{x(x^2+1)} dx = \int \frac{A}{x} dx + \int \frac{Bx+C}{x^2+1} dx$$

$$\left(\frac{x^2+2x}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} \right) \times x(x^2+1)$$

$$x^2+2x = A(x^2+1) + (Bx+C)x$$

$$x^2+2x = Ax^2+A+Bx^2+Cx$$

$$x^2 = Ax^2 + Bx^2 \quad (1) \quad \div x^2 \Rightarrow 1 = A+B$$

$$2x = Cx \quad (2) \quad \div x \Rightarrow C = 2$$

$$0 = A \quad (3) \quad \Rightarrow B = 1$$

$$\therefore \int \frac{x^2+2x}{x(x^2+1)} dx = \int \frac{x+2}{x^2+1} dx$$

$$= \int \frac{x}{x^2+1} dx + \int \frac{2}{x^2+1} dx$$

$$= 2 \ln|x^2+1| + 2 \tan^{-1}x + c$$

$$2. \int \frac{1}{x^2(x^2+4)} dx \quad ; (x^2+4) \text{ is not able analytical}$$

$$\therefore \int \frac{1}{x^2(x^2+4)} dx = \int \frac{A}{x} dx + \int \frac{B}{x^2} dx + \int \frac{Cx+D}{x^2+4} dx$$

$$\left(\frac{1}{x^2(x^2+4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4} \right) * x^2(x^2+4)$$

$$1 = Ax(x^2+4) + B(x^2+4) + (Cx+D)x^2$$

$$1 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + Dx^2$$

$$Ax^3 + Cx^3 = 0 \quad (1) \Rightarrow A = -C$$

$$Bx^2 + Dx^2 = 0 \quad (2) \Rightarrow B = -D$$

$$4Ax = 0 \quad (3) \Rightarrow A = 0 \Rightarrow C = 0$$

$$4B = 1 \quad (4) \Rightarrow B = \frac{1}{4} ; D = -\frac{1}{4}$$

$$\therefore \int \frac{1}{x^2(x^2+4)} dx = \int \frac{\frac{1}{4}}{x^2} dx - \int \frac{\frac{1}{4}}{x^2+4} dx$$

$$= \frac{1}{4} \int x^{-2} dx - \frac{1}{4} \int \frac{1}{x^2+4} dx$$

$$= \frac{1}{4} \cdot \frac{1}{x} - \frac{1}{4} \cdot \frac{1}{2} \tan^{-1} \frac{x}{2} + C$$

$$= \frac{1}{4x} - \frac{1}{8} \tan^{-1} \frac{x}{2} + C$$