6. $\lim_{x \to 2} \sqrt{x^2 - 1}$

Soly

$$\lim_{x \to 2} \sqrt{x^2 - 1} = \sqrt{2^2 - 1} = \sqrt{3}$$

So that the limit of the function $\lim_{x \to 2} \sqrt{x^2 - 1}$ when x approaches 2 is $\sqrt{3}$.

2. The limits at Zero

In this type of limit, we take a value to the right of Zero and the same value to the left of Zero, and we substitute it into the limit, If the limit on the right is the same as the limit of the left, then it is said that the limit is located at Zero

But if the limit on the right side is not equal to the limit on the left side, the the limit does not exist at Zero.

Ex Find the limit of the..

1.

$$\lim_{x \to 0} |x| = \begin{cases} \lim_{x \to -0} |x| = |1| = 1 \\ \lim_{x \to +0} |x| = |-1| = 1 \end{cases}$$

∴ The left Side = The right Side

The limit is exist at Zero to $|x|$

2. $\lim_{x \to 0} x$

Soly $\lim_{x \to +0} x = 1$ and $\lim_{x \to -0} x = -1$

The left side ≠ The right side

∴ The limit is not exist at Zero to x

3. The limit at ∞

we first limit at the following theorem:-

Theorem:-

$$\lim_{x\to\infty} \frac{a}{x} = 0 \quad ; \; a : \text{any real no.}$$

$$\frac{\text{constant}}{\infty} = 0$$

Ex Find the limit of:

1. $\lim_{x\to\infty} \frac{x^2+4}{2x^2+5x}$

بالقسمة على أبراس

$$= \lim_{x\to\infty} \frac{\frac{x^2}{x^2}+\frac{4}{x^2}}{\frac{2x^2}{x^2}+\frac{5x}{x^2}}$$

$$= \lim_{x\to\infty} \frac{1+\frac{4}{x^2}}{2+\frac{5}{x}}$$

$$= \frac{1+\frac{4}{\infty}}{2+\frac{5}{\infty}} = \frac{1+0}{2+0} = \frac{1}{2}$$

1. The real number

The limit approaches L when x approaches to a:

That is:

$$\lim_{x \to a} f(x) = L$$

If the value of x approaches any real number, then the Substitution is made directly with the Same Value

This is : $f(x)\ x \to a = L$

Ex1. Find $\lim_{x \to 2} x^3 + 4x$

Solu $\lim_{x \to 2} x^3 + 4x$.

$$= (2)^3 + 4(2) = 8 + 8 = 16$$

So that the limit of the function $\lim_{x \to 2} x^3 + 4x$ when x approaches 2 is 16

2. $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

١. التعويض

Solu

$$= \frac{4-4}{2-2} = \frac{0}{0}$$ كيفية غير معرفة $\frac{0}{1} = 0$ $\frac{1}{0} = \infty$

٢. التبسيط

$$\therefore \lim_{x \to 2} \frac{(x-2)(x+2)}{x-2}$$

$$= \lim_{x \to 2} (x+2) = 4$$