The Properties of the limits:-

let c, n, m are constants.

1. $\lim_{x \to a} k = k$

2. $\lim_{x \to a} (f(x) \mp g(x)) = \lim_{x \to a} f(x) \mp \lim_{x \to a} g(x)$

3. $\lim_{x \to a} (f(x) * g(x)) = \lim_{x \to a} f(x) * \lim_{x \to a} g(x)$

4. $\lim_{x \to a} \sqrt{f(x)} = \sqrt{\lim_{x \to a} f(x)}$

5. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$

6. $\lim_{x \to a} c f(x) = c \lim_{x \to a} f(x)$

7. $\lim_{x \to a} (f(x))^{\frac{n}{m}} = (\lim_{x \to a} f(x))^{\frac{n}{m}}$

Ex

1. $\lim_{x \to 2} 5 = 5$

2. $\lim_{x \to -1} (2x + 1) = \lim_{x \to -1} 2x + \lim_{x \to -1} 1 = -2 + 1 = -1$

3. $\lim_{x \to 4} \sqrt{x} = \sqrt{\lim_{x \to 4} x} = \sqrt{4} = 2$

4. $\lim_{x \to 2} \frac{1 - x}{1 + x} = \frac{\lim_{x \to 2} (1 - x)}{\lim_{x \to 2} (1 + x)} = \frac{-1}{3}$

Lapital's Rule

In the limits, if the result after Substitution is equal :-

$\frac{\text{Zero}}{\text{Zero}}$, $\frac{0}{0}$ then we derive the numerator alone and the denominator alone, and then Substitute the value of x.

That is, when there is an unknown quantity, we use Lapital's rule to find the derivative of the numerator and denominator separately and find the value of the limit

Ex Using Lapital's rule find the limit..

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

Solu

$$= \frac{3^2 - 9}{3 - 3} = \frac{0}{0}$$ unknown Value

$$\lim_{x \to 3} \frac{2x}{1} = 2(3) = 6$$

OR

$$\lim_{x \to 3} \frac{(x-3)(x+3)}{(x-3)}$$

$$= \lim_{x \to 3} (x+3) = 6$$

H.W.

$$\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$$

Soln

$$= \frac{(3)^3 - 27}{3 - 3} = \frac{0}{0}$$

$$(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$$

$$\therefore \lim_{x \to 3} \frac{x^3 - 27}{x-3} = \lim_{x \to 3} \frac{(x-3)(x^2 + 3x + 9)}{x - 3}$$

$$= \lim_{x \to 3} x^2 + 3x + 9$$

$$= 27$$

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Or

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$$

$$\lim_{x \to a} \frac{x^3 - 3^3}{x - 3} = 3(3)^2 = 27$$