PROBABILITY

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Chapter One
Set Theory

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A review of some of the set theoretical concepts that are relevant to probability is given in this Chapter.

1.1 Basic Set Theory

Definition 1.1.1.

A *set* is a collection. of objects. The objects are called. *elements* or *points* of the set. A set is usually denoted by a capital letter, e.g. A, B, C, ... and so forth. If w is a point or element belonging to the set A, we shall write $w \in A$; if w is not an element of A, we shall write $w \notin A$.

There are two methods for specifying the contents of the set: either by listing its elements or by stating properties which characterizethe elements of the set.

Example 1.1.1

Let

$$A= \{ 0,1,2,3,4\}$$

and

$$B= \{ x:x \text{ is a positive integer and } x < 8\},$$

then

$$B= \{ 1,2,3,4,5,6,7\}$$

Defintion 1.1.2

The set A is said to be a *subset* of B if every element of A is also an element of B; this is denoted by $A \subset B$ or $B \supset A$; read "A is contained in B" or "B contains A", respectively.

The set of all objects under discussion in a given situation is called the *space*, *universe* or *universal set*. We shall call it the space and denote it by S or Ω . All sets under investigation are assumed to be subsets of the space S.

Defintion 1.1.3

A set is said to be an *empty set*, or the *null* set, if it has no elements, and is denoted by ϕ .

For any set A, we have $\phi \subset A \subset S$.

Two sets A and B are said to be *equivalent*, or *equal*, if $A \subset B$ and $B \subset A$, and we write A=B.

Example 1.1.2

Let A= $\{x : x^2 + 1 = 0, x \text{ is a real number}\}$, then $A = \phi$.

Let X= $\{2,3\}$ and $Y = \{y : y^2 - 5y + 6 = 0\}$, then X = Y.

Defintion 1.1.4

The *complement* of the set A with respect to the space S , denoted by A' , A^c or $\bar{A}$, here after we shall use A', is the set af all elements that are in S but not in A, i.e.

$A' = \{x : x \in S \ \textit{and} \ x \notin A\}$ .... (1.1.1)

Let A and B be any two subsets of S . The set of all points in A that are not in B will be denoted by A/B or A-B and is defined as *set difference*.

$A/B = \{x : x \in A \ \textit{and} \ x \notin B\}$ .... (1.1.2)
Note that $A' = S / A$.

Example 1.1.3

Let S be the set of all natural numbers; i.e.
$S = N = \{1, 2, 3, 4, ...\}$
Define: $A = \{x : x \text{ is an even number}, x \in N\}$, and
$B = \{y : y \text{ is a multiple of } 3, y \in N\}$,

then

$A = \{2,4,6,8,10,12,14, \ldots\ldots\ldots\};$
$B = \{3,6,9,12,15,18, \ldots\ldots\ldots\};$
$A' = \{1,3,5,7,9, \ldots\ldots\ldots\};$
$B' = \{1,2,4,5,7,8,10,11, \ldots\ldots\ldots\};$
$A/B = \{2,4,8,10,14,16,20, \ldots\};$
$B/A = \{3,9,15,21,27, \ldots\ldots\ldots\}$

Definition 1.1.5

Let A and B be any two subsets of the space S. The ***union*** of A and B, denoted by $A \bigcup B$, is the set of all elements belonging either to A or to B, including those which belong to both, i.e.

$$A \bigcup B = \{ x : \text{either } x \in A \text{ } or \text{ } x \in B \text{ or in both}\} \qquad (1.1.3)$$

Hence

$$x \in A \bigcup B \text{ if and only if } x \in A \text{ } or \text{ } x \in B \text{ } \ldots\ldots\ldots\ldots \quad (1.1.4)$$

and

$$x \notin A \bigcup B \text{ if and only if } x \notin A \text{ } and \text{ } x \notin B \text{ } \ldots\ldots\ldots\ldots \quad (1.1.5)$$

The *intersection* of A and B, denoted by $A \bigcap B$, is the set of all elements that are in both A and B, i.e.

$$A \bigcap B = \{x : x \in A \text{ } and \text{ } x \in B\} \text{ } \ldots\ldots\ldots\ldots \quad (1.1.6)$$

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$$x \in A \bigcap B \text{ if and only if } x \in A \text{ } and \text{ } x \in B \qquad (1.1.7)$$

and

$$x \notin A \bigcap B \text{ if and only if } x \notin A \text{ } or \text{ } x \notin B \qquad (1.1.8)$$

Two sets A and B are said to be *disjoint* or ***mutually exclusive*** if they have no common elements, i.e.

$$A \bigcap B = \phi \text{ .}$$

Example 1.1.4

Let $S = \{ (x,y) : 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 1\}$, which is read as the collection of all points (x,y) for which $0 \leq x \leq 2$ and $0 \leq y \leq 1$

Define the following sets :