$$A_i \cap A_j = \phi, \text{ for } i \neq j, i, j = 1, 2, \ldots, n$$

We also define unions and intersection of the events $A_1, A_2, \ldots, A_n$ as

$\bigcup_{i=1}^{n} A_i$ = the event of at least one of the A_i occurred;

$\bigcap_{i=1}^{n} A_i$ = the event of all of the A_i occurred.

Similarly we can define $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$.

3.3. Kinds of Probability

There are three basic ways of classifying probability. These three represent rather different conceptual approaches to the study of probability theory, and in fact experts disagree about which approach is the proper one to use. The three kinds are

1. the classical approach
2. the relative frequency approach. and
3. the subjective approach.

1. Classical Approach

Definition 3.3.1. - Classical Probability

If a random experiment, whose sample space is S, can result in n. mutually exclusive and equally likely outcomes (i.e. each outcome has the same chance for occurrence) and if n(A) of these outcomes have an attribute A, then the probability of A, denoted by P(A), is defined by

$$P(A) = \frac{n(A)}{n} = \frac{\text{number of outcomes favourable to A}}{\text{total number of possible outcomes}} \qquad (3.3.1)$$

Classical probability is often called a *prior probability* because, if we keep using examples like tossing a fair coin, tossing a die and drawing a card from an ordinary deck of cards, we can state the answer in advance without conducting a large number of trials.

Example 3.3.1

1. A fair coin is tossed once, then S = {H, T}. Thus the probability of getting a head is $P(H) = \frac{1}{2}$ and the probability of getting a tail is $P(T) = \frac{1}{2}$.

2. When an unbiased die is thrown once, then S = {1,2,3,4,5,6}.
   The six outcomes are mutually exclusive because two or more faces cannot turn up simultaneously, then
   P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6.
   To find the probability of getting an odd number, define A = {1,3,5}.

By (3.3.1) , we have

$$P(A) = \frac{n(A)}{n} = \frac{3}{6} = \frac{1}{2}.$$

From the classical definition of probability of an event A, we have

1. $0 \le P(A) \le 1$, Since $0 \le \frac{n(A)}{n} \le 1$ for every n.

2. If A is a certain event, then P(A) = 1 because n(A) = n. and if A is an impossible event (null event), then P(A)=0 because n(A) = 0.

For example, in tossing an unbiased die, the probability of getting a number = 8, is zero and the probability of getting a number less than 7 is 1.

3. If A and B are two mutually exclusive events, then
   $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$, because

$$P(A \cup B) = \frac{n(A \cup B)}{n} = \frac{n(A) + n(B)}{n} = \frac{n(A)}{n}$$

$$+ \frac{n(B)}{n} = P(A) + P(B). \quad \ldots (3{\cdot}3{\cdot}2)$$

where n(A), n(B) and $n(A \cup B)$ are the numbers of occurrences of the events A, B and (A or B) respectively.

The result (3. 3. 2) can be extended to the case of k mutually exclusive events $A_1, A_2, \ldots A_k$, i. e.

$$P(A_1, A_2, \ldots, \text{or } A_k) = P(A_1 \cup A_2 \quad \ldots \cup A_k) = P\left(\bigcup_{i=1}^{k} A_i\right)$$

$$= \sum_{i=1}^{k} P(A_i).$$

There are some limitations in the classical approach. It is obvious.

1. that the definition of probability must be modified some how when the total number of possible outcomes is infinite; for example, what is the probability of getting an even number if we draw one number from the natural numbers 1,2,3,...? The intuitive answer to this question is 1/2.

2. Suppose that we toss a bias coin, in this case the two possible outcomes of the coin are not equally likely. To find the probability of getting a head, the classical definition leaves us completely helpless here. So we have.

2. Relative Frequency Approach

We may not be able in the classical approach to answer questions like the following:

1. What is the probability that a man would live more than 70 years?