of less than 80%. At $\alpha = 0.05$, test the claim that there is no difference in the proportions of the small and large nursing homes with a resident vaccination rate of less than 80%.

Source: Nancy Arden, Arnold S. Monto, and Suzanne E. Ohmit, "Vaccine Use and the Risk of Outbreaks in a Sample of Nursing Homes During an Influenza Epidemic," *American Journal of Public Health.*

## Solution

Let $\hat{p}_1$ be the proportion of the small nursing homes with a vaccination rate of less than 80% and $\hat{p}_2$ be the proportion of the large nursing homes with a vaccination rate of less than 80%. Then

$$\hat{p}_1 = \frac{X_1}{n_1} = \frac{12}{34} = 0.35 \qquad \text{and} \qquad \hat{p}_2 = \frac{X_2}{n_2} = \frac{17}{24} = 0.71$$

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{12 + 17}{34 + 24} = \frac{29}{58} = 0.5$$

$$\bar{q} = 1 - \bar{p} = 1 - 0.5 = 0.5$$

Now, follow the steps in hypothesis testing.

**Step 1** State the hypotheses and identify the claim.

$H_0: p_1 = p_2$ (claim) and $H_1: p_1 \neq p_2$

**Step 2** Find the critical values. Since $\alpha = 0.05$, the critical values are $+1.96$ and $-1.96$.

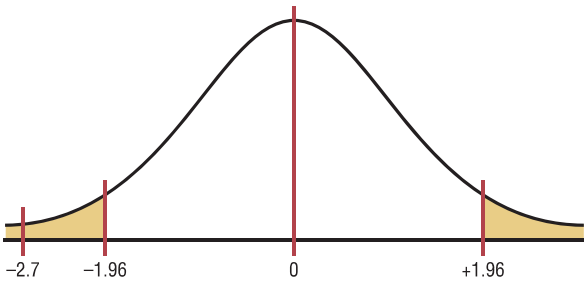
**Step 3** Compute the test value.

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{(0.35 - 0.71) - 0}{\sqrt{(0.5)(0.5)\left(\frac{1}{34} + \frac{1}{24}\right)}} = \frac{-0.36}{0.1333} = -2.7$$

**Step 4** Make the decision. Reject the null hypothesis, since $-2.7 < -1.96$. See Figure 9–8.

**Figure 9–8**
Critical and Test Values for Example 9–9




**Step 5** Summarize the results. There is enough evidence to reject the claim that there is no difference in the proportions of small and large nursing homes with a resident vaccination rate of less than 80%.

## Speaking of Statistics

**Is More Expensive Better?**

An article in the *Journal of the American Medical Association* explained a study done on placebo pain pills. Researchers randomly assigned 82 healthy people to two groups. The individuals in the first group were given sugar pills, but they were told that the pills were a new, fast-acting opioid pain reliever similar to codeine and that they were listed at $2.50 each. The individuals in the other group received the same sugar pills but were told that the pills had been marked down to 10¢ each.

Each group received electrical shocks before and after taking the pills. They were then asked if the pills reduced the pain. Eighty-five percent of the group who were told that the pain pills cost $2.50 said that they were effective, while 61% of the group who received the supposedly discounted pills said that they were effective.

State possible null and alternative hypotheses for this study. What statistical test could be used in this study? What might be the conclusion of the study?

The formula for the confidence interval for the difference between two proportions is shown next.

**Confidence Interval for the Difference Between Two Proportions**

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

**Example 9–11**

Find the 95% confidence interval for the difference of proportions for the data in Example 9–9.

**Solution**

$$\hat{p}_1 = \frac{12}{34} = 0.35 \qquad \hat{q}_1 = 0.65$$

$$\hat{p}_2 = \frac{17}{24} = 0.71 \qquad \hat{q}_2 = 0.29$$

## Exercises 9–4

**1a.** Find the proportions $\hat{p}$ and $\hat{q}$ for each.

*a.* $n = 48, X = 34$ $\hat{p} = \frac{34}{48}, \hat{q} = \frac{14}{48}$
*b.* $n = 75, X = 28$ $\hat{p} = \frac{28}{75}, \hat{q} = \frac{47}{75}$
*c.* $n = 100, X = 50$ $\hat{p} = \frac{50}{100}, \hat{q} = \frac{50}{100}$
*d.* $n = 24, X = 6$ $\hat{p} = \frac{6}{24}, \hat{q} = \frac{18}{24}$
*e.* $n = 144, X = 12$ $\hat{p} = \frac{12}{144}, \hat{q} = \frac{132}{144}$

**1b.** Find each $X$, given $\hat{p}$.

*a.* $\hat{p} = 0.16, n = 100$ 16
*b.* $\hat{p} = 0.08, n = 50$ 4
*c.* $\hat{p} = 6\%, n = 800$ 48
*d.* $\hat{p} = 52\%, n = 200$ 104
*e.* $\hat{p} = 20\%, n = 150$ 30

**2.** Find $\bar{p}$ and $\bar{q}$ for each.

*a.* $X_1 = 60, n_1 = 100, X_2 = 40, n_2 = 100$ $\bar{p} = 0.5; \bar{q} = 0.5$
*b.* $X_1 = 22, n_1 = 50, X_2 = 18, n_2 = 30$ $\bar{p} = 0.5; \bar{q} = 0.5$
*c.* $X_1 = 18, n_1 = 60, X_2 = 20, n_2 = 80$ $\bar{p} = 0.27; \bar{q} = 0.73$
*d.* $X_1 = 5, n_1 = 32, X_2 = 12, n_2 = 48$ $\bar{p} = 0.2125; \bar{q} = 0.7875$
*e.* $X_1 = 12, n_1 = 75, X_2 = 15, n_2 = 50$ $\bar{p} = 0.216; \bar{q} = 0.784$

**For Exercises 3 through 14, perform these steps.**

*a.* State the hypotheses and identify the claim.
*b.* Find the critical value(s).
*c.* Compute the test value.
*d.* Make the decision.
*e.* Summarize the results.

**Use the traditional method of hypothesis testing unless otherwise specified.**

**3. Married People** In a specific year 53.7% of men in the United States were married and 50.3% of women were married. Random samples of 300 men and 300 women found that 178 men and 139 women were married (not necessarily to each other.) At the 0.05 level of significance can it be concluded that the proportion of men who were married is greater than the proportion of women who were married?

Source: *New York Times Almanac.*

**4. Undergraduate Financial Aid** A study is conducted to determine if the percent of women who receive financial aid in undergraduate school is different from the percent of men who receive financial aid in undergraduate school. A random sample of undergraduates revealed these results. At $\alpha = 0.01$, is there significant evidence to reject the null hypothesis?

| | Women | Men |
|---|---|---|
| Sample size | 250 | 300 |
| Number receiving aid | 200 | 180 |

Source: U.S. Department of Education, National Center for Education Statistics.

**5. High School Graduation Rates** The overall U.S. public high school graduation rate is 73.4%. For Pennsylvania it is 83.5% and for Idaho 80.5%—a difference of 3%. Random samples of 1200 students from each state indicated that 980 graduated in Pennsylvania and 940 graduated in Idaho. At the 0.05 level of significance can it be concluded that there is a difference in the proportions of graduating students?

Source: *World Almanac.*

**6. Animal Bites of Postal Workers** In Cleveland, a sample of 73 mail carriers showed that 10 had been bitten by an animal during one week. In Philadelphia, in a sample of 80 mail carriers, 16 had received animal bites. Is there a significant difference in the proportions? Use $\alpha = 0.05$. Find the 95% confidence interval for the difference of the two proportions.

**7. Lecture versus Computer-Assisted Instruction** A survey found that 83% of the men questioned preferred computer-assisted instruction to lecture and 75% of the women preferred computer-assisted instruction to lecture. There were 100 individuals in each sample. At $\alpha = 0.05$, test the claim that there is no difference in the proportion of men and the proportion of women who favor computer-assisted instruction over lecture. Find the 95% confidence interval for the difference of the two proportions.

**8. Leisure Time** In a sample of 50 men, 44 said that they had less leisure time today than they had 10 years ago. In a sample of 50 women, 48 women said that they had less leisure time than they had 10 years ago. At $\alpha = 0.10$ is there a difference in the proportions? Find the 90% confidence interval for the difference of the two proportions. Does the confidence interval contain 0? Give a reason why this information would be of interest to a researcher.

Source: Based on statistics from Market Directory.

**9. Desire to Be Rich** In a sample of 80 Americans, 44 wished that they were rich. In a sample of 90 Europeans, 41 wished that they were rich. At $\alpha = 0.01$, is there a difference in the proportions? Find the 99% confidence interval for the difference of the two proportions.

**10. Seat Belt Use** In a sample of 200 men, 130 said they used seat belts. In a sample of 300 women, 63 said they used seat belts. Test the claim that men are more safety-conscious than women, at $\alpha = 0.01$. Use the *P*-value method.

**11. Dog Ownership** A survey found that in a sample of 75 families, 26 owned dogs. A survey done 15 years ago found that in a sample of 60 families, 26 owned dogs. At $\alpha = 0.05$ has the proportion of dog owners changed over the 15-year period? Find the 95% confidence