



## Hypothesis Testing

### Objectives

After completing this chapter, you should be able to

- 1 Understand the definitions used in hypothesis testing.
- 2 State the null and alternative hypotheses.
- 3 Find critical values for the  $z$  test.
- 4 State the five steps used in hypothesis testing.
- 5 Test means when  $\sigma$  is known, using the  $z$  test.
- 6 Test means when  $\sigma$  is unknown, using the  $t$  test.
- 7 Test proportions, using the  $z$  test.
- 8 Test variances or standard deviations, using the chi-square test.
- 9 Test hypotheses, using confidence intervals.
- 10 Explain the relationship between type I and type II errors and the power of a test.

### Outline

#### Introduction

#### 8-1 Steps in Hypothesis Testing—Traditional Method

#### 8-2 $z$ Test for a Mean

#### 8-3 $t$ Test for a Mean

#### 8-4 $z$ Test for a Proportion

#### 8-5 $\chi^2$ Test for a Variance or Standard Deviation

#### 8-6 Additional Topics Regarding Hypothesis Testing

#### Summary

A claim, though, can be stated as either the null hypothesis or the alternative hypothesis; however, the statistical evidence can only *support* the claim if it is the alternative hypothesis. Statistical evidence can be used to *reject* the claim if the claim is the null hypothesis. These facts are important when you are stating the conclusion of a statistical study.

Table 8-1 shows some common phrases that are used in hypotheses and conjectures, and the corresponding symbols. This table should be helpful in translating verbal conjectures into mathematical symbols.

### Example 8-1

State the null and alternative hypotheses for each conjecture.

- A researcher thinks that if expectant mothers use vitamin pills, the birth weight of the babies will increase. The average birth weight of the population is 8.6 pounds.
- An engineer hypothesizes that the mean number of defects can be decreased in a manufacturing process of compact disks by using robots instead of humans for certain tasks. The mean number of defective disks per 1000 is 18.
- A psychologist feels that playing soft music during a test will change the results of the test. The psychologist is not sure whether the grades will be higher or lower. In the past, the mean of the scores was 73.

### Solution

- $H_0: \mu = 8.6$  and  $H_1: \mu > 8.6$
- $H_0: \mu = 18$  and  $H_1: \mu < 18$
- $H_0: \mu = 73$  and  $H_1: \mu \neq 73$

After stating the hypothesis, the researcher designs the study. The researcher selects the correct *statistical test*, chooses an appropriate *level of significance*, and formulates a plan for conducting the study. In situation A, for instance, the researcher will select a sample of patients who will be given the drug. After allowing a suitable time for the drug to be absorbed, the researcher will measure each person's pulse rate.

Recall that when samples of a specific size are selected from a population, the means of these samples will vary about the population mean, and the distribution of the sample means will be approximately normal when the sample size is 30 or more. (See Section 6-3.) So even if the null hypothesis is true, the mean of the pulse rates of the sample of patients will not, in most cases, be exactly equal to the population mean of 82 beats per minute. There are two possibilities. Either the null hypothesis is true, and the difference between the sample mean and the population mean is due to chance; *or* the null hypothesis is false, and the sample came from a population whose mean is not 82 beats per minute but is some other value that is not known. These situations are shown in Figure 8-1.

The farther away the sample mean is from the population mean, the more evidence there would be for rejecting the null hypothesis. The probability that the sample came from a population whose mean is 82 decreases as the distance or absolute value of the difference between the means increases.

If the mean pulse rate of the sample were, say, 83, the researcher would probably conclude that this difference was due to chance and would not reject the null hypothesis. But if the sample mean were, say, 90, then in all likelihood the researcher would conclude that the medication increased the pulse rate of the users and would reject the null hypothesis. The question is, Where does the researcher draw the line? This decision is not made on feelings or intuition; it is made statistically. That is, the difference must be significant and in all likelihood not due to chance. Here is where the concepts of statistical test and level of significance are used.