$$P(A \cup B) = \frac{n(A \cup B)}{n} = \frac{n(A) + n(B)}{n} = \frac{n(A)}{n}$$

$$+ \frac{n(B)}{n} = P(A) + P(B). \qquad \dots (3{\cdot}3{\cdot}2)$$

where n(A), n(B) and n(A $\cup$ B) are the numbers of occurrences of the events A, B and (A or B) respectively.

The result (3. 3. 2) can be extended to the case of k mutually exclusive events A_1, A_2, ... A_k, i. e.

$$P(A_1, A_2, \dots, \text{or } A_k) = P(A_1 \cup A_2 \quad \dots \cup A_k) = P\left(\bigcup_{i=1}^{k} A_i\right)$$

$$= \sum_{i=1}^{k} P(A_i).$$

There are some limitations in the classical approach. It is obvious.

1. that the definition of probability must be modified some how when the total number of possible outcomes is infinite; for example, what is the probability of getting an even number if we draw one number from the natural numbers 1,2,3,...? The intuitive answer to this question is 1/2.

2. Suppose that we toss a bias coin, in this case the two possible outcomes of the coin are not equally likely. To find the probability of getting a head, the classical definition leaves us completely helpless here. So we have.

2. Relative Frequency Approach

We may not be able in the classical approach to answer questions like the following:

1. What is the probability that a man would live more than 70 years?

2. What is the probability that student chosen at random from a class of 100 students will pass the exam?

Without experimentation, we may not be able to do this in advance. So another kind of approch may be more useful. It is the *relative frequency* or *Posterior probability*, which defines the probability as either :

1. The observed relative frequency of an event in a very large number of trials,

or

2. The proportion of times that an event occurs in the long run when the conditions of performing the experiment under it are stable.

If the number of independent trials is infinite, then we define the probability of an event E as

$$P(E) = \lim_{n \to \infty} \frac{h}{n}$$

where h and n denote the number of occurrences of E and the number of independent trials, respectively. The ratio $\frac{h}{n}$ is called the *relative frequency* of E.

Example 3.3.2.

1. A bolt is drawn at random from a box containing 600 bolts of which 42 are defective. Then the probability that the bolt is good is

$$P(\text{good}) = \frac{h}{n} = \frac{558}{600} = 0.93$$

where h denotes the number of good bolts.

2. Suppose the experiment consists of sampling electric bulbs from a large collection of bulbs. We shall postulate that the probability of a given bulb being defective is p. We can estimate p by selecting several bulbs at random

from the collection and calculating the relative frequency of the defective bulbs.

Example 3.3.3

1. The probability of getting an even number if we draw one number from the natural numbers 1,2,3,...., is.

$$P(\text{even}) = \lim_{n \to \infty} \frac{n}{2n} = \frac{1}{2},$$

where n is the number of the even integers 2,4,6,... In the sâme way, the probability of an odd number is

$$P(\text{odd}) = \frac{1}{2}.$$

2. A fair coin is tossed n times (n=100, 1000, 10000, and 100000) and the number of heads appearing is observed. The results are recorded in Table 3.1 , below.

Table 3.1

No. of trials n	Observed no. of heads h	Observed relative frequency $\frac{h}{n}$	long run expected relative frequen
100	56	0.56	0.5
1000	512	0.512	0.5
10000	4996	0.4996	0.5
100000	49998	0.49998	0.5

We can say that the relative frequency becomes stable as the number of tosses becomes large (if we do the experiment under uniform conditions).

3. Subjective Probability

Subjective probability is based on the personal beliefs of the person making the probability assessment. In fact, *subjective probability* can be defined as the probability assigned to an event by an individual , based on whether ,evidence is available. This evidence may be in the form of relative frequencies of past occurrences, or it may be just an educated guess.

Example 3.3.4.

1. Suppose an insurance company knows from past acturial data that of all 40-year old males, 60 out of 10000 will die within a one-year period. The company can estimate the probability of death for that age group as

$$P\text{ (death)} = \frac{60}{10000} = 0.006.$$

2. A business man may feel that the odds for the success of a new restaurant are 3 to 2. This means that he would be willing to bet 300 I.D. against 200 I.D., or 3000 I.D. against 2000 I.D. that the restaurant will succeed. In this way he expresses the strength of his belief regarding the uncertainties connected with the success of the new restaurant.
The probability of success of a new restaurant is

$$\frac{3}{3+2} = \frac{3}{5} = 0{\cdot}6$$

3.4. Problems

1. Toss a fair coin 3 times . Write down the sample space S. Define the following events
   A = (2 heads)
   B = (at least one tail)
   C = (at least two heads)
   Find the probabilities of (i) A or B , (ii) not A, (iii) A and B, (iv) A and C and (v) C/B.

2. Throw a biased coin twice (see Fig. 3.1) below. Define
   E_1 = {at least one head}
   and
   E_2 = {tail on first toss}
   Find $P(E_1)$, $P(E_2)$ and $P(E_1 E_2)$.

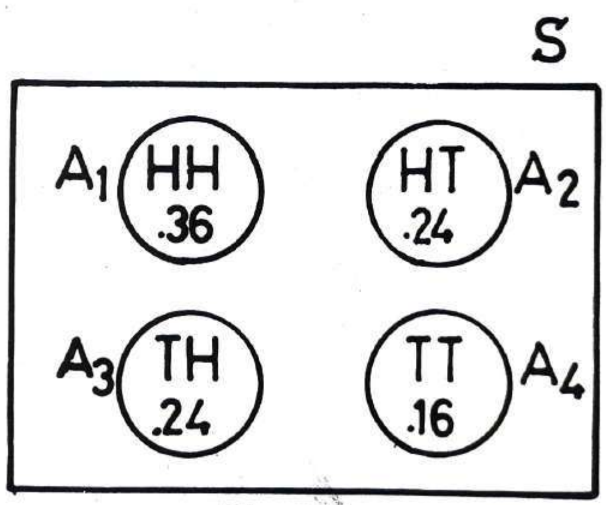

Fig. 3.1

3. An experiment involves choosing an integer N between O and 9 inclusive. Let A= $\{N \leq 5\}$, B=$\{3 \leq N \leq 7\}$, C={N is even}. List the points that belong to the following events.
   $A \cap B \cap C$, $A \cup (B \cap C)$, $(A \cup B) \cap C'$, $(A \cap B) \cap (A \cup C)'$

4. A pair of unbiased dice are rolled once What is the probability that the sum of the upturned faces will equal 7?

5. A card is selected at random from an ordinary deck of 52 cards. Let.

A = {the card is a spade}

and B = {the card is a picture card, i.e. a jack, queen or king}

Find P (A), P (B) and P (A ∩ B).

6. Determine the probability of each event :

i. an odd number appears in the toss of a fair die,

ii. a king appears. in drawing a. single card from an ordinary deck of 52 cards;

iii. at most one tail appears in the toss of three fair coins ;

iv. a white. marble appears in drawing a marble from a box containing 6 white, 3 red and 5 black marables.

7. Two digits are chosen at random from the digits 1,2,3,4,5. List the 2-digit numbers that can be formed. What is the probability that the number is even ?

8. A coin is weighted so that head is twice as likely to apper as tail. Find P(H).

9. A university class contains 3 times as many male students as female students. One student is chosen at random, What is the probability that he will be a male? a female?

10. Throw 2 fair coins 50 times and count the number of heads and compare your results with the theoretical ones. Construct a table of your results in the following form.

No. of heads : 0 1 2

No. of times :