Measuring Cyclical Variations
قياس التغيرات الدورية

Cyclical changes are defined as long-term movements that repeatedly rise and fall along the general trend line of a phenomenon's time series. Economic factors are among the most important causes of cyclical changes in the time series of economic or commercial phenomena. These types of changes are called economic cycles, as the successive time periods of economic phenomena reflect the periods of economic recession or economic prosperity that characterize the economies of some countries.

تعرف التغيرات الدورية بانها التحركات طويلة الامد التي تتكرر صعوداً ونزولًا على خط الاتجاه العام للسلسلة الزمنية لظاهرة ما. وان من اهم الاسباب التي تؤدي الى حدوث التغيرات الدورية في السلاسل الزمنية للظواهر ذات الطابع الاقتصادي او التجاري هي الاسباب الاقتصادية مما يطلق على هذا النوع من التغيرات بالدورات الاقتصادية إذ تعكس الفترات الزمنية المتعاقبة للظواهر الاقتصادية حلات الكساد او الرفاه الاقتصادي التي تتصف بها اقتصادات بعض الدول.

To estimate cyclical changes and separate them from the rest of the time series components, follow the following steps:

1- Estimate the general trend line equation using the least squares method, and ensure that the sum of the time series is not equal to zero.

تقدير معادلة خط الاتجاه العام باستخدام طريقة المربعات الصغرى وان مجموع تسلسل الزمن لايساوي صفراً.

2- Calculate the trend values of the phenomenon (ŷ) based on the general trend line equation.

حساب القيم الاتجاهية للظاهرة (ŷ) اعتماداً على معادلة خط الاتجاه العام.

3- Remove the observations of the phenomenon (y) from the effect of the general trend according to the following formula:

تخليص مشاهدات الظاهرة (y) من اثر الاتجاه العام وفقاً للصيغة الاتية:

$$y^* = \frac{y}{\hat{y} = T} \times 100\%$$

4- Finding the adjusted seasonal indicators (%S) using the ratio method to the general trend according to the following law:

ايجاد المؤشرات الموسمية المعدلة ($\%S$) باستخدام طريقة النسبة الى الاتجاه العام وفقاً للقانون الاتي:

$$S\% = \frac{\bar{Q}_i}{\sum_{i=1}^{4} \bar{Q}_i} \times 4 \times 100$$

5− Finding the periodic ratios according to the following law:

ايجاد النسب الدورية وفق القانون الاتي:

$$C\% = \frac{y^*}{S} \times 100$$

6- Extricating observations of the phenomenon (y) from the effect of periodic changes according to the following formula:

تخليص مشاهدات الظاهرة (y) من اثر التغيرات الدورية وفقاً للصيغة الاتية:

$$y^{***} = \frac{y}{C} \times 100\%$$

Example: The following data represents the production quantity (in thousands of tons) of a particular crop during the period 2000-2002.

Requirements:

1- Find the equation of the general trend line using the least squares method.
2- Excluding the effect of the general trend from observations of the phenomenon.
3- Calculate seasonal indices (%S) using the ratio to the general trend method.
4- Calculate the periodic ratios of production quantities for the above period.
5- Removing the effect of periodic changes from observations of the phenomenon (y).

Years	2000				2001				2002			
seasons	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4
productio	20	26	18	21	24	30	29	28	31	28	20	25

Sol:

1- Finding the equation of the general trend line.

Years	seasons	y	t	t*y	t^2	$\hat{y}_i = 22.01 + 0.46t_i$	$y^* = \frac{y}{\hat{y}} \times 100$
2002	**Q_1**	20	1	20	1	22.47	(20/22.47)×100 =89.01
	Q_2	26	2	52	4	22.93	(26/22.93)×100 =113.39
	Q_3	18	3	54	9	23.39	(18/23.39)×100 =76.96
	Q_4	21	4	84	16	23.85	(21/23.85)×100 =88.05
2003	**Q_1**	24	5	120	25	24.31	(24/24.31)×100 =98.72
	Q_2	30	6	180	36	24.77	(30/24.77)×100 =121.11
	Q_3	29	7	203	49	25.23	(29/25.23)×100 =114.94
	Q_4	28	8	224	64	25.69	(28/25.69)×100 =108.99
2004	**Q_1**	31	9	279	81	26.15	(31/26.15)×100 =118.55
	Q_2	28	10	280	100	26.61	(28/26.61)×100 =105.22
	Q_3	20	11	220	121	27.07	(20/27.07)×100 =73.88
	Q_4	25	12	300	144	27.53	(25/27.53)×100 =90.81
sum		**300**	**78**	**2016**	**650**		

$$\hat{y} = \hat{a} + \hat{b}t$$

$$\hat{b} = \frac{n\sum_{i=1}^{n} y_i t_i - \sum_{i=1}^{n} y_i \sum_{i=1}^{n} t_i}{n\sum_{i=1}^{n} t_i^2 - \left(\sum_{i=1}^{n} t_i\right)^2} = \frac{12 \times (2016) - (300)(78)}{12 \times (650) - (78)^2} = 0.46$$

$$\hat{a} = \frac{\sum_{i=1}^{n} y_i}{n} - b\frac{\sum_{i=1}^{n} t_i}{n} = \frac{300}{12} - 0.46\frac{78}{12} = 25 - (0.46)(6.5) = 22.01$$

$$\hat{y}_i = 22.01 + 0.46t_i$$

2- The effect of the general trend is excluded from the observations of the phenomenon by first finding the directional values of the phenomenon ($\hat{y}_i$) by substituting the order of the seasons ($t_i = 1,2,3,4,\dots,12$) in the estimated general trend equation, then calculating the observations of the phenomenon stripped of the effect of the general trend according to the following formula:

$$y^* = \frac{\mathbf{y}}{\hat{\mathbf{y}}} \times 100$$

3- Calculating seasonal indicators adjusted for the seasons according to the method of ratio to the general trend. This is done by first organizing the ratios of production values devoid of the effect of the general trend in a second table in order to be able to calculate averages for the seasons.

Years/Seasons	Q1	Q2	Q3	Q4
2002	89.01	113.39	76.96	88.05
2003	98.72	121.11	114.94	108.99
2004	118.55	105.22	73.88	90.81
Sum of Seasons	306.28	339.72	265.78	287.85
$\bar{Q}_i$	102.09	113.24	88.59	95.95
$\sum_{i=1}^{4} \bar{Q}_i$	399.87			
	102.12 %	113.28 %	88.62 %	95.98 %

Then calculate the seasonal indicators according to the following relationship:

$$S\% = \frac{\bar{Q}_i}{\sum_{i=1}^{4} \bar{Q}_i} \times 4 \times 100$$

$$S_1\% = \frac{\bar{Q}_1}{\sum_{i=1}^{4} \bar{Q}_i} \times 4 \times 100 = \frac{102.09}{399.87} \times 4 \times 100 = 102.12\%$$

$$S_2\% = \frac{\bar{Q}_2}{\sum_{i=1}^{4} \bar{Q}_i} \times 4 \times 100 = \frac{113.24}{399.87} \times 4 \times 100 = 113.28\%$$

$$S_3\% = \frac{\bar{Q}_3}{\sum_{i=1}^{4} \bar{Q}_i} \times 4 \times 100 = \frac{88.59}{399.87} \times 4 \times 100 = 88.62\%$$

$$S_4\% = \frac{\bar{Q}_4}{\sum_{i=1}^{4} \bar{Q}_i} \times 4 \times 100 = \frac{95.95}{399.87} \times 4 \times 100 = 95.98\%$$

4- The periodic ratios are found and their effect is removed from the observations according to the following two laws:

$$C = \frac{y^*}{S\%} \times 100$$

Years	seasons	y	y^*	%S	$C = \frac{y^*}{S\%} \times 100$	y^{***}
2000	Q_1	20	89.01	102.12	(89.01/102.12)×100=87.16	(20/87.12)×100=22.95
	Q_2	26	113.39	113.28	(113.39/113.28)×100=100.10	(26/100.10)×100 =25.97
	Q_3	18	76.96	88.62	(76.96/88.62)×100 =86.84	(18/86.84)×100 =20.73
	Q_4	21	88.05	95.98	(88.05/95.98)×100 =91.74	(21/91.74)×100 =22.89
2001	Q_1	24	98.72	102.12	(98.72/102.12)×100 =96.67	(24/96.67)×100 =24.83
	Q_2	30	121.11	113.28	(121.11/113.28)×100=106.91	(30/106.91)×100=28.06
	Q_3	29	114.94	88.62	(114.94/88.62)×100=129.70	(29/129.70)×100=22.36
	Q_4	28	108.99	95.98	(108.99/95.98)×100=113.55	(28/113.55)×100=24.66
2002	Q_1	31	118.55	102.12	(118.55/102.12)×100=116.09	(31/116.09)×100=26.70
	Q_2	28	105.22	113.28	(105.22/113.28)×100=92.88	(28/92.88)×100 =30.15
	Q_3	20	73.88	88.62	(73.88/88.62)×100 =83.37	(20/83.37)×100 =23.99
	Q_4	25	90.81	95.98	(90.81/95.98)×100 =94.61	(25/94.61)×100 =26.42