Biostatistics – Spring 2026
Lecture 09: Actuarial Life Tables

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Introduction

In the previous lecture, we studied standard life tables for population survival by age.

In many clinical studies, however, we do not follow people by age. Instead, we follow them over study time such as months or years after diagnosis or treatment.

Also, some subjects leave the study early. This creates **censoring**.

For such data, we use the **actuarial life table** method.

By the end of this lecture, you should be able to:

- explain what an actuarial life table is,
- divide follow-up time into intervals,
- adjust for censored observations,
- compute interval death probability and interval survival probability,
- compute cumulative survival.

1. Why Do We Need the Actuarial Life Table?

Sometimes exact event times are not available, or we choose to summarize follow-up into intervals such as:

- 0–6 months,
- 6–12 months,
- 12–18 months,
- 18–24 months.

In each interval, we record:

- number at risk,
- number of events,
- number censored.

This method is especially useful when follow-up is grouped into intervals rather than exact event times.

2. Main Symbols in the Actuarial Life Table

For interval i, we define:

- n_i: number at risk at the beginning of interval i,
- d_i: number of events in interval i,
- c_i: number censored in interval i,
- N_i: effective number at risk in interval i,
- q_i: probability of event in interval i,
- p_i: probability of survival in interval i,
- $S(t_i)$: cumulative survival up to the end of interval i.

3. Effective Number at Risk

Because censored subjects usually leave at different times inside the interval, a common approximation is that, on average, they contribute half of the interval.

Therefore, the effective number at risk is:

$$N_i = n_i - \frac{c_i}{2}.$$

This is one of the most important ideas in the actuarial method.

4. Interval Event Probability and Survival Probability

The interval probability of event is:

$$q_i = \frac{d_i}{N_i}.$$

The interval probability of survival is:

$$p_i = 1 - q_i.$$

Then the cumulative survival is computed recursively by:

$$S(t_i) = S(t_{i-1}) \times p_i, \qquad S(0) = 1.$$

5. Worked Example

Suppose 100 patients are followed after liver transplantation for 24 months. The follow-up is divided into 4 intervals.

Interval (months)	d_i	c_i
0–6	10	5
6–12	8	4
12–18	7	3
18–24	5	2

Step 1: Compute n_i

At the beginning:

$$n_1 = 100.$$

Then:

$$n_2 = 100 - 10 - 5 = 85$$
$$n_3 = 85 - 8 - 4 = 73$$
$$n_4 = 73 - 7 - 3 = 63.$$

Step 2: Compute N_i

$$N_1 = 100 - \frac{5}{2} = 97.5$$
$$N_2 = 85 - \frac{4}{2} = 83$$
$$N_3 = 73 - \frac{3}{2} = 71.5$$
$$N_4 = 63 - \frac{2}{2} = 62.$$

Step 3: Compute q_i and p_i

$$q_1 = \frac{10}{97.5} = 0.1026, \qquad p_1 = 0.8974$$
$$q_2 = \frac{8}{83} = 0.0964, \qquad p_2 = 0.9036$$
$$q_3 = \frac{7}{71.5} = 0.0979, \qquad p_3 = 0.9021$$
$$q_4 = \frac{5}{62} = 0.0806, \qquad p_4 = 0.9194.$$

Step 4: Compute cumulative survival

$$S(t_1) = 1 \times 0.8974 = 0.8974$$
$$S(t_2) = 0.8974 \times 0.9036 = 0.8109$$
$$S(t_3) = 0.8109 \times 0.9021 = 0.7315$$
$$S(t_4) = 0.7315 \times 0.9194 = 0.6725.$$

6. Final Actuarial Table

Interval	n_i	c_i	N_i	d_i	p_i	$S(t_i)$
0–6	100	5	97.5	10	0.8974	0.8974
6–12	85	4	83	8	0.9036	0.8109
12–18	73	3	71.5	7	0.9021	0.7315
18–24	63	2	62	5	0.9194	0.6725

7. Interpretation

The cumulative survival at 24 months is approximately:

$$S(24) \approx 0.6725.$$

Interpretation:

The estimated probability of surviving beyond 24 months is about 67.25%.

8. When Is the Actuarial Method Useful?

It is useful when:

- exact event times are not available,
- follow-up is grouped into intervals,
- a simple interval-based summary is needed.

9. Common Student Mistakes

- Using n_i instead of N_i in the denominator.
- Forgetting to subtract half of the censored observations.
- Confusing interval survival p_i with cumulative survival $S(t_i)$.
- Using addition instead of multiplication for cumulative survival.

10. Summary

- The actuarial life table uses grouped time intervals.
- Censored observations are adjusted by using

$$N_i = n_i - \frac{c_i}{2}.$$

- The interval event probability is

$$q_i = \frac{d_i}{N_i}.$$

- Cumulative survival is obtained by multiplying interval survival probabilities.

Homework (HW)

HW1

A study starts with 80 subjects and the following data are observed:

Interval	d_i	c_i
0–4	6	4
4–8	5	3
8–12	4	2

Compute:

1. n_i
2. N_i
3. q_i
4. p_i
5. $S(t_i)$

HW2

Explain why we use

$$N_i = n_i - \frac{c_i}{2}$$

instead of just n_i.

HW3

If $S(12) = 0.74$, interpret this result in words.