Finding *P*-values for the *F* test statistic is somewhat more complicated since it requires looking through all the *F* tables (Table H in Appendix C) using the specific d.f.N. and d.f.D. values. For example, suppose that a certain test has $F = 3.58$, d.f.N. $= 5$, and d.f.D. $= 10$. To find the *P*-value interval for $F = 3.58$, you must first find the corresponding *F* values for d.f.N. $= 5$ and d.f.D. $= 10$ for α equal to 0.005, 0.01, 0.025, 0.05, and 0.10 in Table H. Then make a table as shown.

α	0.10	0.05	0.025	0.01	0.005
F	2.52	3.33	4.24	5.64	6.87

Now locate the two *F* values that the test value 3.58 falls between. In this case, 3.58 falls between 3.33 and 4.24, corresponding to 0.05 and 0.025. Hence, the *P*-value for a right-tailed test for $F = 3.58$ falls between 0.025 and 0.05 (that is, $0.025 < P\text{-value} < 0.05$). For a right-tailed test, then, you would reject the null hypothesis at $\alpha = 0.05$ but not at $\alpha = 0.01$. The *P*-value obtained from a calculator is 0.0408. Remember that for a two-tailed test the values found in Table H for α must be doubled. In this case, $0.05 < P\text{-value} < 0.10$ for $F = 3.58$.

Once you understand the concept, you can dispense with making a table as shown and find the *P*-value directly from Table H.

Example 9–16 Airport Passengers

The CEO of an airport hypothesizes that the variance in the number of passengers for American airports is greater than the variance in the number of passengers for foreign airports. At $\alpha = 0.10$, is there enough evidence to support the hypothesis? The data in millions of passengers per year are shown for selected airports. Use the *P*-value method. Assume the variable is normally distributed.

American airports		Foreign airports	
36.8	73.5	60.7	51.2
72.4	61.2	42.7	38.6
60.5	40.1		

Source: Airports Council International.

Solution

Step 1 State the hypotheses and identify the claim.

H_0: $\sigma_1^2 = \sigma_2^2$ and H_1: $\sigma_1^2 > \sigma_2^2$ (claim)

Step 2 Compute the test value. Using the formula in Chapter 3 or a calculator, find the variance for each group.

$s_1^2 = 246.38$ and $s_2^2 = 95.87$

Substitute in the formula and solve.

$$F = \frac{s_1^2}{s_2^2} = \frac{246.38}{95.87} = 2.57$$

Step 3 Find the *P*-value in Table H, using d.f.N. $= 5$ and d.f.D. $= 3$.

α	0.10	0.05	0.025	0.01	0.005
F	5.31	9.01	14.88	28.24	45.39

Since 2.57 is less than 5.31, the *P*-value is greater than 0.10. (The *P*-value obtained from a calculator is 0.234.)

5. Using Table H, find the critical value for each.

a. Sample 1: $s_1^2 = 128$, $n_1 = 23$
Sample 2: $s_2^2 = 162$, $n_2 = 16$
Two-tailed, $\alpha = 0.01$

b. Sample 1: $s_1^2 = 37$, $n_1 = 14$
Sample 2: $s_2^2 = 89$, $n_2 = 25$
Right-tailed, $\alpha = 0.01$

c. Sample 1: $s_1^2 = 232$, $n_1 = 30$
Sample 2: $s_2^2 = 387$, $n_2 = 46$
Two-tailed, $\alpha = 0.05$

d. Sample 1: $s_1^2 = 164$, $n_1 = 21$
Sample 2: $s_2^2 = 53$, $n_2 = 17$
Two-tailed, $\alpha = 0.10$

e. Sample 1: $s_1^2 = 92.8$, $n_1 = 11$
Sample 2: $s_2^2 = 43.6$, $n_2 = 11$
Right-tailed, $\alpha = 0.05$

6. (ans) Using Table H, find the *P*-value interval for each *F* test value.

a. $F = 2.97$, d.f.N. = 9, d.f.D. = 14, right-tailed
b. $F = 3.32$, d.f.N. = 6, d.f.D. = 12, two-tailed
c. $F = 2.28$, d.f.N. = 12, d.f.D. = 20, right-tailed
d. $F = 3.51$, d.f.N. = 12, d.f.D. = 21, right-tailed
e. $F = 4.07$, d.f.N. = 6, d.f.D. = 10, two-tailed
f. $F = 1.65$, d.f.N. = 19, d.f.D. = 28, right-tailed
g. $F = 1.77$, d.f.N. = 28, d.f.D. = 28, right-tailed
h. $F = 7.29$, d.f.N. = 5, d.f.D. = 8, two-tailed

For Exercises 7 through 20, perform the following steps. Assume that all variables are normally distributed.

a. State the hypotheses and identify the claim.
b. Find the critical value.
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

7. Ages of Hospital Patients The average age of hospital inpatients has gradually increased to 52.5 years. Studies of two major health care systems found the following information. At the 0.05 level of significance is there sufficient evidence to conclude a difference between the two variances?

	System 1	System 2
Sample size	60	60
Sample mean	49.8	50.2
Sample standard deviation	5.4	7.6

Source: *New York Times Almanac.*

8. Museum Attendance A metropolitan children's museum open year-round wants to see if the variance in daily attendance differs between the summer and winter months. Random samples of 30 days each were selected and showed that in the winter months, the sample mean daily attendance was 300 with a standard deviation of 52, and the sample mean daily attendance for the summer months was 280 with a standard deviation of 65. At $\alpha = 0.05$ can we conclude a difference in variances?

9. Wolf Pack Pups Does the variance in average number of pups per pack differ between Montana and Idaho wolf packs? Random samples of packs were selected for each area, and the numbers of pups per pack were recorded. At the 0.05 level of significance, can a difference in variances be concluded?

Montana	4	3	5	6	1	2	8	2
wolf packs	3	1	7	6	5			
Idaho	2	4	5	4	2	4	6	3
wolf packs	1	4	2	1				

Source: www.fws.gov

10. Noise Levels in Hospitals In a hospital study, it was found that the standard deviation of the sound levels from 20 areas designated as "casualty doors" was 4.1 dBA and the standard deviation of 24 areas designated as operating theaters was 7.5 dBA. At $\alpha = 0.05$, can you substantiate the claim that there is a difference in the standard deviations?

Source: M. Bayo, A. Garcia, and A. Garcia, "Noise Levels in an Urban Hospital and Workers' Subjective Responses," *Archives of Environmental Health.*

11. Calories in Ice Cream The numbers of calories contained in $\frac{1}{2}$-cup servings of randomly selected flavors of ice cream from two national brands are listed here. At the 0.05 level of significance, is there sufficient evidence to conclude that the variance in the number of calories differs between the two brands?

Brand A		Brand B	
330	300	280	310
310	350	300	370
270	380	250	300
310	300	290	310

Source: *The Doctor's Pocket Calorie, Fat and Carbohydrate Counter.*

12. Winter Temperatures A random sample of daily high temperatures in January and February is listed below. At $\alpha = 0.05$ can it be concluded that there is a difference in variances in high temperature between the two months?

Jan.	31	31	38	24	24	42	22	43	35	42
Feb.	31	29	24	30	28	24	27	34	27	

13. Population and Area Cities were randomly selected from the list of the 50 largest cities in the United States (based on population). The areas of each

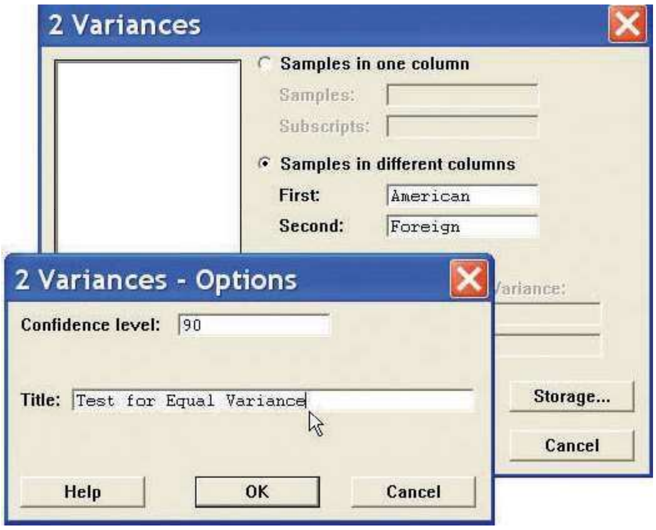
Technology *Step by Step*

MINITAB
Step by Step

Test for the Difference Between Two Variances

For Example 9–16, test the hypothesis that the variance in the number of passengers for American and foreign airports is different. Use the *P*-value approach.

American airports	Foreign airports
36.8	60.7
72.4	42.7
60.5	51.2
73.5	38.6
61.2	
40.1	



1. Enter the data into two columns of MINITAB.
2. Name the columns American and Foreign.
   a) Select **Stat>Basic Statistics>2-Variances.**
   b) Click the button for Samples in different columns.
   c) Click in the text box for First, then double-click C1 American.
   d) Double-click C2 Foreign, then click on [Options]. The dialog box is shown. Change the confidence level to **90** and type an appropriate title. In this dialog, we cannot specify a left- or right-tailed test.
3. Click [OK] twice. A graph window will open that includes a small window that says $F = 2.57$ and the *P*-value is 0.437. Divide this two-tailed *P*-value by 2 for a one-tailed test.

There is not enough evidence in the sample to conclude there is greater variance in the number of passengers in American airports compared to foreign airports.

TI-83 Plus or TI-84 Plus
Step by Step

Hypothesis Test for the Difference Between Two Variances (Data)

1. Enter the data values into L_1 and L_2.
2. Press **STAT** and move the cursor to TESTS.