Summary

Many times researchers are interested in comparing two parameters such as two means, two proportions, or two variances. These measures are obtained from two samples, then compared using a z test, t test, or an F test.

- If two sample means are compared, when the samples are independent and the population standard deviations are known, a z test is used. If the sample sizes are less than 30, the populations should be normally distributed. (9–1)
- If two means are compared when the samples are independent and the sample standard deviations are used, then a t test is used. Both variances are assumed to be unequal. (9–2)
- When the two samples are dependent or related, such as using the same subjects and comparing the means of before and after tests, then the t test for dependent samples is used. (9–3)
- Two proportions can be compared by using the z test for proportions. In this case, each of n_1p_1, n_1q_1, n_2p_2, and n_2q_2 must all be 5 or more. (9–4)
- Two variances can be compared by using an F test. The critical values for the F test are obtained from the F distribution. (9–5)
- Confidence intervals for differences between two parameters can also be found.

Important Terms

Important Formulas

Formula for the z test for comparing two means from independent populations; σ_1 and σ_2 are known:

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

Formula for the confidence interval for difference of two means when σ_1 and σ_2 are known:

$$(\bar{X}_1 - \bar{X}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Formula for the t test for comparing two means (independent samples, variances not equal), σ_1 and σ_2 unknown:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

and d.f. = the smaller of $n_1 - 1$ or $n_2 - 1$.

Formula for the confidence interval for the difference of two means (independent samples, variances unequal), σ_1 and σ_2 unknown:

$$(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

and d.f. = smaller of $n_1 - 1$ and $n_2 - 2$.

Formula for the t test for comparing two means from dependent samples:

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}}$$

where $\bar{D}$ is the mean of the differences

$$\bar{D} = \frac{\Sigma D}{n}$$

and s_D is the standard deviation of the differences

$$s_D = \sqrt{\frac{n\Sigma D^2 - (\Sigma D)^2}{n(n-1)}}$$

Formula for confidence interval for the mean of the difference for dependent samples:

$$\bar{D} - t_{\alpha/2}\frac{s_D}{\sqrt{n}} < \mu_D < \bar{D} + t_{\alpha/2}\frac{s_D}{\sqrt{n}}$$

and d.f. = $n - 1$.

East					West				
495	390	540	445	420	525	400	310	375	750
410	550	499	500	550	390	795	554	450	370
389	350	450	530	350	385	395	425	500	550
375	690	325	350	799	380	400	450	365	425
475	295	350	485	625	375	360	425	400	475
275	450	440	425	675	400	475	430	410	450
625	390	485	550	650	425	450	620	500	400
685	385	450	550	425	295	350	300	360	400

Source: *Pittsburgh Post-Gazette.*

16. **Prices of Low-Calorie Foods** The average price of a sample of 12 bottles of diet salad dressing taken from different stores is \$1.43. The standard deviation is \$0.09. The average price of a sample of 16 low-calorie frozen desserts is \$1.03. The standard deviation is \$0.10. At $\alpha = 0.01$, is there a significant difference in price? Find the 99% confidence interval of the difference in the means.

17. **Jet Ski Accidents** The data shown represent the number of accidents people had when using jet skis and other types of wet bikes. At $\alpha = 0.05$, can it be concluded that the average number of accidents per year has increased from one period to the next?

1987–1991			1992–1996		
376	650	844	1650	2236	3002
1162	1513		4028	4010	

Source: *USA TODAY.*

18. **Salaries of Chemists** A sample of 12 chemists from Washington state shows an average salary of \$39,420 with a standard deviation of \$1659, while a sample of 26 chemists from New Mexico has an average salary of \$30,215 with a standard deviation of \$4116. Is there a significant difference between the two states in chemists' salaries at $\alpha = 0.02$? Find the 98% confidence interval of the difference in the means.

19. **Family Incomes** The average income of 15 families who reside in a large metropolitan East Coast city is \$62,456. The standard deviation is \$9652. The average income of 11 families who reside in a rural area of the Midwest is \$60,213, with a standard deviation of \$2009. At $\alpha = 0.05$, can it be concluded that the families who live in the cities have a higher income than those who live in the rural areas? Use the *P*-value method.

20. **Mathematical Skills** In an effort to improve the mathematical skills of 10 students, a teacher provides a weekly 1-hour tutoring session for the students. A pretest is given before the sessions, and a posttest is given after. The results are shown here. At $\alpha = 0.01$, can it be concluded that the sessions help to improve the students' mathematical skills?

Student	1	2	3	4	5	6	7	8	9	10
Pretest	82	76	91	62	81	67	71	69	80	85
Posttest	88	80	98	80	80	73	74	78	85	93

21. **Egg Production** To increase egg production, a farmer decided to increase the amount of time the lights in his hen house were on. Ten hens were selected, and the number of eggs each produced was recorded. After one week of lengthened light time, the same hens were monitored again. The data are given here. At $\alpha = 0.05$, can it be concluded that the increased light time increased egg production?

Hen	1	2	3	4	5	6	7	8	9	10
Before	4	3	8	7	6	4	9	7	6	5
After	6	5	9	7	4	5	10	6	9	6

22. **Factory Worker Literacy Rates** In a sample of 80 workers from a factory in city A, it was found that 5% were unable to read, while in a sample of 50 workers in city B, 8% were unable to read. Can it be concluded that there is a difference in the proportions of nonreaders in the two cities? Use $\alpha = 0.10$. Find the 90% confidence interval for the difference of the two proportions.

23. **Male Head of Household** A recent survey of 200 households showed that 8 had a single male as the head of household. Forty years ago, a survey of 200 households showed that 6 had a single male as the head of household. At $\alpha = 0.05$, can it be concluded that the proportion has changed? Find the 95% confidence interval of the difference of the two proportions. Does the confidence interval contain 0? Why is this important to know?

Source: Based on data from the U.S. Census Bureau.

24. **Money Spent on Road Repair** A politician wishes to compare the variances of the amount of money spent for road repair in two different counties. The data are given here. At $\alpha = 0.05$, is there a significant difference in the variances of the amounts spent in the two counties? Use the *P*-value method.

County A	County B
$s_1 = \$11,596$	$s_2 = \$14,837$
$n_1 = 15$	$n_2 = 18$

25. **Heights of Basketball Players** A researcher wants to compare the variances of the heights (in inches) of four-year college basketball players with those of players in junior colleges. A sample of 30 players from each type of school is selected, and the variances of the heights for each type are 2.43 and 3.15, respectively. At $\alpha = 0.10$, is there a significant difference between the variances of the heights in the two types of schools?