Biostatistics – Spring 2026
Lecture 12: Measures of Association
Risk, Relative Risk, Odds, and Odds Ratio

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Introduction

In epidemiology and biostatistics, we often want to measure the association between an exposure and an outcome.

For example:

- smoking and lung cancer,
- obesity and diabetes,
- a new drug and recovery,
- occupational exposure and disease.

Testing association with Chi-square tells us whether an association may exist, but it does not tell us the **strength** or **direction** of that association.

For that purpose, we use measures such as:

- risk,
- relative risk (RR),
- odds,
- odds ratio (OR).

By the end of this lecture, you should be able to:

- organize exposure–disease data in a 2×2 table,
- compute risk in each group,
- compute and interpret relative risk,
- compute odds in each group,
- compute and interpret the odds ratio,
- distinguish when RR and OR are usually used.

1. The Basic 2×2 Table

Many epidemiologic data can be arranged as follows:

	Disease Yes	Disease No	Total
Exposure Yes	a	b	$a+b$
Exposure No	c	d	$c+d$
Total	$a+c$	$b+d$	N

Here:

- a: exposed and diseased,
- b: exposed and not diseased,
- c: unexposed and diseased,
- d: unexposed and not diseased.

2. Risk in Each Group

The risk of disease in the exposed group is:

$$\text{Risk}_{\text{exp}} = \frac{a}{a+b}$$

The risk of disease in the unexposed group is:

$$\text{Risk}_{\text{unexp}} = \frac{c}{c+d}$$

These are simply proportions of diseased individuals in each group.

3. Relative Risk (RR)

Relative risk compares the risk in the exposed group to the risk in the unexposed group.

$$RR = \frac{a/(a+b)}{c/(c+d)}$$

Interpretation

- If $RR > 1$, exposure is associated with higher risk.
- If $RR < 1$, exposure may be protective.
- If $RR = 1$, there is no difference in risk between groups.

Example 1: Smoking and lung cancer

	Lung cancer Yes	Lung cancer No	Total
Smoking Yes	50	150	200
Smoking No	20	180	200
Total	70	330	400

Risk among smokers:

$$\frac{50}{200} = 0.25$$

Risk among non-smokers:

$$\frac{20}{200} = 0.10$$

So:

$$RR = \frac{0.25}{0.10} = 2.5$$

Interpretation

Smokers have 2.5 times the risk of lung cancer compared with non-smokers.

Example 2: Gender and heart disease

	Heart disease Yes	Heart disease No	Total
Female	3	147	150
Male	8	192	200
Total	11	339	350

Risk among females:

$$\frac{3}{150} = 0.02$$

Risk among males:

$$\frac{8}{200} = 0.04$$

So:

$$RR = \frac{0.02}{0.04} = 0.5$$

Interpretation

In this sample, females have half the risk of heart disease compared with males.

4. Odds

Odds compare the probability of an event to the probability of no event.

If the probability of disease is p, then the odds of disease are:

$$\text{Odds} = \frac{p}{1-p}$$

In a 2×2 table:

$$\text{Odds in exposed group} = \frac{a}{b}$$

$$\text{Odds in unexposed group} = \frac{c}{d}$$

Example: odds of lung cancer

Using the smoking example:

$$\text{Odds among smokers} = \frac{50}{150} = 0.333$$

$$\text{Odds among non-smokers} = \frac{20}{180} = 0.111$$

Interpretation: The disease odds are higher among smokers.

5. Odds Ratio (OR)

The odds ratio compares the odds in one group to the odds in another group.

$$OR = \frac{a/b}{c/d} = \frac{ad}{bc}$$

Interpretation

- If $OR > 1$, exposure is associated with higher odds of disease.
- If $OR < 1$, exposure may be protective.
- If $OR = 1$, there is no association in terms of odds.

Example: smoking and lung cancer

$$OR = \frac{50 \times 180}{150 \times 20} = \frac{9000}{3000} = 3$$

Interpretation

The odds of lung cancer among smokers are 3 times the odds among non-smokers.

6. Relative Risk Versus Odds Ratio

Although RR and OR are related, they are not the same.

Relative Risk

- compares risks,
- easier to interpret directly,
- commonly used in cohort studies and clinical follow-up studies.

Odds Ratio

- compares odds,
- commonly used in case-control studies,
- also widely used in logistic regression.

Important note

When disease is rare, RR and OR may be numerically close. But when disease is common, OR may be farther from 1 than RR.

7. When Do We Use Each Measure?

Use RR mainly when:

- you can directly compute risk,
- the study is cohort-based,
- subjects are followed over time.

Use OR mainly when:

- the study is case-control,
- risk cannot be directly estimated from sampling design,
- logistic models are used.

8. Common Interpretation Rules

- $RR = 2$: the exposed group has twice the risk.
- $RR = 0.5$: the exposed group has half the risk.
- $OR = 3$: the odds are three times as large.
- Value $= 1$: no association.

But always remember:

A large RR or OR in a sample does not automatically mean statistical significance. Confidence intervals are also important.

9. Common Student Mistakes

- Confusing risk with odds.
- Interpreting OR as if it were exactly RR.
- Forgetting the cross-product formula $OR = \frac{ad}{bc}$.
- Using RR in case-control studies without thinking about design.
- Saying "cause" when the study only shows association.

10. Summary

- A 2×2 table is the basic structure for exposure–outcome data.
- Relative risk compares risks between groups.
- Odds compare probability of disease to probability of no disease.
- Odds ratio compares odds between groups.
- RR is commonly used in cohort studies; OR is commonly used in case-control studies.

Homework (HW)

HW1

For the following table, compute the risk in each group and the relative risk:

	Disease Yes	Disease No	Total
Exposed	24	76	100
Unexposed	12	88	100

HW2

For the same data in HW1, compute:

1. the odds in each group,
2. the odds ratio.

HW3

Explain in words the difference between RR and OR.

HW4

In what type of study is OR usually preferred over RR?